Mr. William Oughtred's
K̄
KEY
OF THE
Mathematicks.

Newly Tranſlated from the beſt EDITION,

With NOTES,

Rendring it Eaſy and Intelligible to
leſs Skilful *Readers*.

In which alſo,

Some PROBLEMS

Left Unanſwer'd by the Author are Reſolv'd.

Abſolutely neceſſary

For all *Gagers*, *Surveyors*, *Gunners*, *Military Officers*, *Mariners*, &c.

Recommended by Mr. E. HALLEY,
Fellow of the Royal Society.

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TO THE
READER.

THE *Clavis Mathematica* of Mr. *William Oughtred*, is a Book of ſo eſtabliſhed a Reputation, that it were needleſs to ſay any thing thereof. It was formerly Tranſlated by Dr. *Wood* into *Engliſh*; but from an Edition which has been ſince much bettered and augmented; and beſides, the conciſe Brevity of the Author is ſuch, as in many places to need an Explication, to render it Intelligible to the leſs knowing in Mathematical matters. This Tranſlation is New and from the fulleſt Edition, and may be of good Uſe to all Beginners in the Analytical Art, eſpecially to ſuch, who tho they may be Ignorant of the *Latin* Tongue, may yet be deſirous to inform themſelves in Geometry: and to all ſuch I recommend it as a very uſeful Treatiſe.

E. Halley.

ERRATA.

PAGE 3. Line 9. for x read X. line 12. *idem.* p. 4 Sect. 9. *more* or *mo.* r. *plus* or *pl.* ſect. 10. for *leſs* or *le.* r. *minus* or *mi.* p. 4. line 29. r. *accommodate.* p. 26. l. 5. for zω r. z-ω. p. 46. l. 11. for AA. r. AC. p. 65. l. 21. for vq4 r. rc4. p. 86. l. 29. for *Wherefote* r. *Therefore.* p. 110. l. 6. for *inſerting* r. *inverting.* p. 111. l. 9. for x r.——.

Mr. *Oughtred's*

M^{r} OUGHTRED's KEY of the MATHEMATICKS, WITH NOTES.

CHAP. I.

Of NOTATION.

1. THE following Table is very uſeful, not only with reſpect to the Notation of Numbers, which it does at firſt ſight exhibit; but alſo all Computation by Numbers, Common, and Figurate, and alſo Artificial, which are called *Logarithms*.

Integers. **Fractions;**

9	8	7	6	5	4	3	2	1	0	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{6}$	$\bar{7}$	$\bar{8}$	$\bar{9}$	
M	M	M	M	M	M	M	C	X	I	X	C	M	M	M	M	M	M	M	
M	M	M	M	C	X	I						I	X	C	M	M	M	M	&c.
M	C	X	I												I	X	C	M	

2. In this Table the upper Numbers are Indexes or Exponents of the lower, which are Terms in continual proportion from Unity on both ſides; and are in Integers, or whole Numbers, affirmative, in Parts, or Fractions negative. And the proportion is decuple towards the left hand, and ſubdecuple towards the right; as the ſubſcribed numeral Letters do ſhew. The Progreſſion, I ſay, is from Unity in *Integers*, ſuch as this, 1, 10, 100, 1000, 10000: and in *Decimal Parts*, as, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$, *&c.*

3. And after this manner, in any Progreſſion, Indexes are to be ſet to the terms increaſing or decreaſing from Unity in any Ratio or Proportion.

4. I have indeed formed this Table in decuple proportion, that the value of any Numbers may be eſtimated by Steps and Periods; and alſo becauſe this Decimal way of reckoning is much eaſier than that ſexagenary way (or by 60's) in Aſtronomical Computations. This he was ſenſible of, who firſt

reduced the Canon of Sines from the Semidiameter of 60 to 1, with noughts annexed.

5. Decimal Parts are written in the ſame line with Integers; but are diſtinguiſhed by a rectangular line; which is therefore called the Separating Line. And as in Integers every term is increaſed by a continual Multiplication by × according to its diſtance from Unity. So in the Parts, every Term is diſtinguiſhed by a Diviſion, by the product of a continual Multiplication of × firſt upon Unity, and then upon each thus reſulting Term, according to its diſtance from Unity, denoted by the Index or Exponent.

6. Decimal Parts take their denomination from the place of their laſt Figure, as 0|5 are five tenths, 0|56 are fifty ſix hundredths, and ſo onwards.

7. Circles or Noughts before Integers or after Decimals ſignifie nothing; but after Integers and before Decimals, they ſhew us the diſtance of a Figure from Unity either way, according to which we are to judge of its value, as 0005 ſignifie no more than 5; and 0|500 are five tenths.

8. Wherefore in writing Decimal Parts, let the Separating line be always made,

and the empty places, if there be any, be filled with Noughts, as 0 | 00005 are five hundred thouſandth Parts.

9. The Sign of Addition is + more, or *mo.*

10. The Sign of Subſtraction is — leſs, or *le.*

11. The Sign belongs to the Quantity before which it is placed. And a Quantity with no Sign muſt be underſtood to be affirmative, or to have the Sign +, although it be not expreſſed.

12. And obſerve, That I uſe the Signs + and — when one ſingle Magnitude is affirmed or denied of another ſingle Magnitude; but the Signs *pl.* and *mi.* when a compounded Magnitude or Quantity is affirmed or denied of a ſingle, or a ſingle of a compounded.

13. Quantities may be denoted either by Numbers that ſignifie their meaſure or by Symbols; as a line 7 Inches long by 7 or by a Letter, as A; or B, or C, *&c.* or by two Letters ſet at each end of the Line, AB, BC, CD, *&c.* at pleaſure; ſo that you remember for what Quantity every Symbol ſtands.

14. This Symbolical Arithmetick is much more ccaomodate to Analyticks (in which you take that which is ſought after as if al-

ready known, and ſo find, according to the Rules of that Art, what it really is) than Numerous Arithmetick: in the laſt the Numbers do ſo ſweep away one another, that there remain no footſteps of the ſeveral Operations: in the former the Symbols remain without any Alteration, and you may not only with pleaſure view every ſtep in the whole proceſs, but you alſo gain Theorem for the ſolution of the like Queſtions in other Quantities.

NOTES.

1. *In the fifteenth Chapter you may ſee what he means by a* Figurate Number.

2. The Progreſſion is decuple] i. e. *In it you multiply each preceding Term by* 10 *to produce the next.* Subdecuple] i. e. *You divide by* 10 *here as you multiply by* 10 *there.*

6. Decimal parts, *&c.*] *Though indeed you may without error call* $0\underline{|56}$, 5 *tenths and* 6 *hundredths; yet ſince* $\frac{50}{100} = \frac{5}{10}$, *therefore 'tis beſt to call them both by the ſame denomination,* 56 *hundredths.*

CHAP. II.

Of ADDITION.

1. THE Number which ariſes from *Addition* is called the Summ or Aggreagate, as 3 and 7 make 10.

2. *Addition* begins at the right hand, and ſets down the particular Products of the ſeveral rows underneath in their proper places.

3. All the given Numbers together are equal to the Summ.

Examples of Addition.

			l.	*s:*	*d.*
79403	3794	236			
8956	584	3	17	13	4
67293	947	08	9	16	7
5087	4720	7439	238	09	6
160739	48	5	70	00	10
			48	10	3
	10094	8599	384	10	9

4. Symbolical Addition joins all the given Magnitudes without changing their Signs.

to

to	$3A$	A	$5A$	$3A$	A
add	A	$-A$	$-3A$	$-5A$	E
Sum.	$3A+A$	$A-A$	$3A-5A$	$5A-3A$	$A+E$
i. e.	$4A$	0	$-2A$	$2A$	

to	$A+B$	$A+B$
add	$A-B$	$A-C$
Sum.	$2A$	$2A+B-C$

So in the Addition of *Index's*	3	$\overline{3}$
	$\overline{2}$	2
	1	$\overline{1}$

NOTES.

4. *The Meaning of* $-A$, *is, that there is a Defect or* Subduction *of* A; *and consequently* $+A-A=0$: *for that Affirmative* A *being put where there was leſs than nothing by* A, *does no more than ſupply that Defect.*

CHAP. III.

Of SUBDUCTION.

1. THE Number found by *Subduction* is called the Remainder or the Difference or Exceſs: as, out of 7 take 3 there remain 4.

2. *Subduction* begins at the right hand, and writes the Differences of the Figures of each place in their proper places.

3. In *Subduction* the Number to be ſubducted together with the Difference, is equal to the Number from which the Subduction is to be made.

EXAMPLES.

$$\begin{array}{r} 347206836 \\ 6807592 \\ \hline 340399244 \end{array} \qquad \begin{array}{r|l} 3794 & 236 \\ 947 & 08 \\ \hline 2847 & 156 \end{array} \qquad \begin{array}{rrr} l. & s. & d. \\ 17 & 13 & 4 \\ 9 & 16 & 7 \\ \hline 7 & 16 & 9 \end{array}$$

4. *Symbolical Subduction* joins the Magnitudes given, changing the Signs of thoſe which are to be ſubducted.

out of	4A	3A	5A	A
take	A	5A	−3A	E
rem.	4A−A	3A−5A	5A+3A	A−E
i. e.	3A	−2A	8A	

out of	A	A
take	B+C	B−C
rem.	A−B−C	A−E+C

So in *Indexes.* $\left\{ \begin{array}{rr} \bar{3} & 3 \\ 2 & \bar{2} \\ \hline 5 & \bar{5} \end{array} \right.$

NOTES.

NOTES.

4. *As to the* Subduction *of one Quantity from another when they have like Signs, either + or —, there is no difficulty. As to the* Subduction *of one Quantity from another when they have unlike Signs, either an affirmative Quantity is ſuducted from a negative, or a negative from an affirmative, it may be explained thus,*

Take —A *from* +A *the remainder will be* 2A *: for as to ſubduct a poſitive Quantity from* any Totum *is to turn it into a Defect; ſo to ſubduct a Defect from* any Totum *is to add ſo much. The caſe is this, you are to ſuppoſe any Quantity, ſuppoſe* A, *with a defect of* E, *or* —E *lying by it, to take away that defect is to ſupply it, and conſequently the Summ that thus ariſes will be* A+E. *As for the other 'tis now very eaſie.*

CHAP. IV.

Of MULTIPLICATION.

1. THE Number found by *Multiplication* is called the *Factus*, or the Product, or a Rectangle, or a Plane: for one of the ſuppoſed Numbers may be called the Longitude the other the Latitude; and both may

be called Factors and Sides; the greateſt power of any two Quantities being a Figure compoſed by them, whoſe angles are right, and whoſe ſides are parallel.

2. *Multiplication* begins at the right hand, and multiplies all the Figures of one given Number upon all the Figures of the other, and collecteth the ſeveral Products into one Summ, a regard being had to their places. If Decimal parts be mingled with the Numbers to be multiplied, you muſt cut off from the Product as many places of parts as there are in both the Factors: For in *Multiplication* the Index of the Product of any two Figures is found by adding the Indexes of the two multiplied Figures. So $58|\underline{73}$ multiplied by 600 makes 35238. For the Index of the Figure 6 in 600 is 2; the Index of the laſt Figure 3 in $58|\underline{73}$ is $\overline{2}$. Let the Indexes $\overline{2}$ and 2 be added, there will ariſe 0 for the Index of the laſt Figure in the Product 35238; which therefore belongs to the place of Unites. And ſo of any other Figures.

3. If there be Noughts in the Numbers to be multiplied, omit them, and multiply the Figures, and then in the Product let there be added as many places of Integers as you omitted Noughts in both the Factors.

4. In *Multiplication.* As 1 is to either of the Factors, ſo is the other to the Product. As 4 upon 6 makes 24, therefore as,

14 :: 6 . 24.
16 :: 4 . 24.

EXAMPLES.

```
   4576        580|34     358        5873
    892         47|5         600      600
   ----      ---------   --------   -----
   9152                  214800     35238
  41184        290 170
 36608        4062 38
 -------     23213 6
 4081792     27566|150
```

5. There is a very uſeful Contraction of *Multiplication* according to the following Method.

If it ſuffice to have your Product not entire, but wanting ſome of the laſt Figures; you muſt put the place of Unity of the leſſer Number, under that Figure of the greater, whoſe Index is equal to the Number of Figures, either to be cut off *in the Integers,* or to be left *in the Parts.* And then ſet the reſt of the Figures of the leſſer Number under the greater in the inverſe order. And let each Figure of the Multiplier begin to multiply that of the Multiplicand, which is juſt over; but ſo as to have reſpect to what would have been brought thither from the following places.

Of this Abridgement *there are Four* Caſes.

```
246|914
 72|53
-------
 74 07
 12 35
    49
    17
-------
 87 08
```

Caſ. I. If you would have the Product without any Parts. Set the place of Unity of the leſſer under the place of Unity in the greater: as in this Example, where 246|914 upon 35|27 produces 8708 Integers; all the decimal parts being dropt.

```
914|246
     72|53
-----------
740 742 00
123 457 00
  4 938 28
  1 728 40
-----------
870 8|65 68
```

Caſe. II. If you would have the Product with ſome places of parts; ſuppoſe 4: ſet the place of Unity of the leſſer Number under the Fourth place of the Parts of the greater.

Caſ. III. If you would have the product wanting ſome places of Integers; ſuppoſe

5:

5: ſet the Unites place of the leſſer in the fifth place before the Unites place of the greater. As ſuppoſe 80902 the Sine of 54 degrees, be to be multiplied by 39875 the Sine of the greateſt Declitnaion 23° 30′. The product will be the Declination of the Sun, at the 24° of ♉.

$$\begin{array}{r} 80902 \\ \underline{57893} \\ 24271 \\ 7281 \\ 647 \\ 57 \\ \underline{4} \\ 32260 \end{array}$$

Caſ. IV. If you would have the Product robbed of ſome places of Integers, ſuppoſe 5, to be repair'd again with ſome places of Parts, ſuppoſe 4; becauſe 5—4=1. Therefore you muſt put the Unites place of the leſſer Number one place before the Unites place of the greater. As in this Example, where the Sine 42262 is multiplied by 0|0064; ſo that the five laſt Figures being cut off, four places of parts are reſtored; the Product will be 0|0027.

$$\begin{array}{r} 422\ 62 \\ \underline{4\ 600}|0 \\ \hline 25 \\ \underline{2} \\ 0|\underline{002\ 7.} \end{array}$$

6. *Symbolical Multiplication* connects both the Magnitudes with the Note × or for the moſt part without it, if the Quantities be

denoted with one Letter. If the Signs are both alike, the Product will be affirmative, if unlike, negative.

Note, That A *upon* A, *or* A×A, *or* AA, *is* Aq. AAA, *or* AqA, *is* Ac. AAAA, *or* AqAq, *or* AcA, *is* Aqq, *&c.*

For every ſuperiour Power is made of two inferior, whoſe Dimenſions both together are equal to the number of the Dimenſions of the ſuperiour. And as many as are the Magnitudes which are multiplied, ſo many are the Dimenſions of their Product.

	A	A+E	A+E+I	A—E
upon	E	B	Z	B
makes	AE	BA+BE	ZA+ZE+ZI	BA−BE

B+I
A
BA+A

	3A	AE	A E	A+E
upon	2A	A	A E	A+E
makes	6Aq	AqE	AqAq	Aq+AE
				+AE +Eq
				Aq+2AE+Eq

A+E

$$\begin{array}{ll} & A+E \\ \text{upon} & A-E \\ \hline \text{makes} & AA+AE \\ & \qquad -AE-Eq \\ \hline & Aq-Eq \end{array}$$

If the Side $AB+CD$ be multiplied upon it ſelf, this Square will be produced,

$$BAq+2AB\times CD+CDq.$$

NOTES.

2. And if Decimal parts be added, *&c.*] *Here I ſhall firſt explain that Rule, which he here gives us concerning* Multiplication, *and ſhew then from that Rule that you muſt cut off as many decimal places in the Product, as there are ſuch in both the Factors.*

The RULE *is this.*

The Index of every particular Product is found by adding the Indexes of the Factors.

The Reaſon of this will appear upon viewing the forementioned Table, Cap. 1. *but moſt plainly if we ſet it in this Form.*

&c.

$$\begin{array}{llllll} \text{\&c.} & 4 & 3 & 2 & 1 & 0 \\ \text{\&c.} & \mathrm{XXXX}\,. & \mathrm{XXX}\,. & \mathrm{XX}\,. & \mathrm{X}\,. & \mathrm{I} \end{array}$$

$$\begin{array}{lllll} \bar{1} & \bar{2} & \bar{3} & \bar{4} & \text{\&c.} \\ \frac{\mathrm{I}}{\mathrm{X}}\,. & \frac{\mathrm{I}}{\mathrm{XX}}\,. & \frac{\mathrm{I}}{\mathrm{XXX}}\,. & \frac{\mathrm{I}}{\mathrm{XXXX}} & \text{\&c.} \end{array}$$

As for Integers, 'tis plain that if you take any two or more of the Terms towards the right hand that the Indexes of their Product will be equal to both and all their Indexes added together: take theſe,

$$\begin{array}{ccccc} 2 & + & 3 & = & 5 \\ \mathrm{XX} & \times & \mathrm{XXX} & = & \mathrm{XXXXX}. \end{array}$$

and ſo of any others.

As for Fractions the caſe is the ſame.

$$\begin{array}{ccccc} \bar{2} & + & \bar{3} & = & \bar{5}\,. \\ \frac{\mathrm{I}}{\mathrm{XX}} & \times & \frac{\mathrm{I}}{\mathrm{XXX}} & = & \frac{\mathrm{I}}{\mathrm{XXXXX}}. \end{array}$$

As for Fractions upon Integers: Becauſe Diviſion *undoes what is done by* Multiplication, *therefore,*

$$\overset{3}{XXX} \times \frac{\overset{\bar{2}}{I}}{XX} = \frac{XXX}{XX} = X.$$ *whoſe Index is* 1 = 3−2.

Hence it follows, that you muſt cut off as many places of Parts in the Product, as there are ſuch in both the Factors.

For take any Example. Suppoſe 34|$\underline{45}$ *to be multiply'd by* 3.|$\underline{75}$. *By the Rule the Index of* 5 *in* 1 *is* $\bar{2}$, *and in the other alſo* $\bar{2}$; *which added together make* 4 *the Index of the Product of theſe two: and* 4 *is alſo the number of Parts in both Factors.*

Thus in any other Example.

5. *The Rule in this Paragraph I will not Tranſcribe, but ſuppoſing the Reader to have it in his memory, I will here give him the Reaſon of it; and conſequently of all the Operations in the Four following Caſes. 'Tis this;*

Firſt of all, by this Method 'tis moſt plain, that you place your Unity ſo, that from its particular Product or Line you cut off or ſecure juſt as many Figures as you pleaſe. If you would cut off 5 *in the Integers, you ſet it under the* 5*th place of Integers, and multiply it upon the Figure which is over; and in the Parts, if you* *would ſecure five places there, you ſet the Unity under the fifth place of Parts, and do th like.*

Farther, by placing the Figures of your Multiplicator in the inverſe Order, and multiplying as you are here directed, you will in each rank that is made, by multiplying each Figure of the Multiplicator upon the Multiplicand, effect your deſign. The reaſon is, becauſe when the Figures are ſo placed, the Index of each Figure added to the Index of that which ſtands directly over it, upon which 'tis to be multiplied in the firſt place, are, and muſt be equal to the added Indexes of your Unite, and the Figure which ſtands directly over it.

Let us take the Firſt Example.

```
 246|914          246|914
  35|27            72|53

  17|28398        7407
  49|3828         1235
1234|570            49
7407|42             17
                  ----
                  8708.
```

Where the Indexes of 3 *and* 9 *are equal to the Indexes of* 5 *and* 6, *and equal to the Indexes of* 2 *and* 4, *and* 7 *and* 2; *ſo that the products of all theſe muſt have the ſame place.*

I have here ſet the ſame Sum multiplied the common Way, and drawn a Line, that cuts off thoſe Figures which you drop in this way of Multiplication; where you may ſee, that according to this method of Contraction you multiply the laſt Figure firſt, and the laſt ſave one next, and ſo onwards, and all along drop thoſe Figures which ſtand in thoſe places which you deſign to cut off. And ſo of any other Example.

CHAP. V.

Of DIVISION.

1. THE Number found by *Diviſion* is called the *Quotient*, or alſo the *Parabola*, becauſe it ariſes out of the Application of a plain Number to a given Longitude, that a congruous Latitude may be found. If Two Numbers ſtand one above another, with a Line drawn between them, 'tis as much as to ſay, that the upper is to be divided by the under; as $\frac{12}{4}$, and $\frac{5}{12}$.

2. *Diviſion* begins at the Left Hand, and after it has diſtinguiſhed ſo much of the Dividend as is ſufficient for the Diviſor, and ſet the Diviſor under it, or ſuppoſed it to be written under, it takes the Diviſor out of thoſe diſtinguiſhed Figures as often as it can: Then

the Diviſor being multiplied by the *Quotient*, and the Product taken out of the Dividend it ſets the Diviſor a ſtep forward, and proſecutes the *Diviſion.* Every particular *Quotient* found belongs to that place to which that Figure of the Dividend does, which ſtands, or is ſuppoſed to ſtand over the Unites place of the Diviſor. For in *Diviſion* the Index of every particular Figure of the *Quotient* is found, by taking the Index of the Diviſor from the Index of the Dividend. So $17|\underline{14}$ divided by 857 give $0|\underline{2}$ to the *Quotient*: For the Index of the firſt Figure of the Dividend 17, is 1; and the Index of the firſt Figure of the Diviſor 8, is 2. $1-2=\bar{1}$, for the Index of the firſt Figure; which therefore belongs to the firſt place of Decimals.

3. If the Diviſor have Noughts toward the right, neglect them, and cut of ſo many Figures in the Dividend, and then divide. But in the end of the Diviſion your Noughts and Figures are to be reſtored.

4. In *Diviſion* as the Diviſor is to Unity, ſo is the Dividend to the *Quotient*; or as the Dividend is to the Diviſor, ſo is the *Quotient* to Unity. If you divide 24 by 6 the *Quotient* is 4. 'Tis therefore 61 :: 24.4. and 24.6 :: 4 1.

5. If a quantity be made by the Multiplication of two others, either of them will be its Diviſor, and the other the *Quotient*.

6. In *Multiplication* and *Diviſion* Unity makes no alteration.

7. If a number be multiplied by another, and the Product divided by the ſame, nothing is done: For what *Multiplication* does, *Diviſion* undoes. Wherefore in the application of one magnitude to another if the ſame Magnitude be both above and below the Line let it be blotted out in both places.

EXAMPLES.

297) 187135075 (630084 $\frac{127}{297}$.
~~297~~.
1782
893
~~297~~
891
2507
~~297~~
2376
1315
~~927~~
1188
127

6000) 4320 | 765
720 | 1275

12
8921317
297) 187135075 (630084 $\frac{127}{297}$
178213768
892118

580|34) 27566|150 (47|5
23213680
406237
2901

8. Sometimes a number is to be divided by an irrational or indefinite Number, either conſiſting of Integers or mixt. In this Caſe take as many of the firſt Figures of the Diviſor as are neceſſary, for the firſt Diviſor, and then in every following particular Diviſion drop one of the Figures of the Diviſor towards the Left Hand, till you have got a competent *Quotient.*

EXAMPLE.

17
303
2803
109930
357|0926425) 467023 (1307|80
357093
107127
2500
286

This is an excellent Contraction of *Diviſion*; and of great Uſe in Aſtronomical Computations. For Example; If 137638 be to divide 126223 multiplied by the Total Sine, that is, with five Noughts ſet after the laſt Figure. Set down but one Nought, and for the reſt diminiſh the Diviſor. 137638) 162230)61707.

9. *Symbolical Diviſion* puts the Diviſor under the Dividend with a Line between them, and then conſiders, whether, in their Compoſition, any one magnitude multiplieth both of them, and ſtrikes it out in both.

Diviſion, when the Diviſor and Dividend have the ſame Signs, whether + or — will have + in the *Quotient*, when divers Signs —.

Divide	AE	BAc	BA+A	BA—CA	6 Aq
by	A	Aq	A	B—C	3 A
Quotient	E	BA	B+1	A	2 A.

NOTES.

2. *As for the Rule about Indexes in* Diviſion, *he that will ſee what has been ſaid concerning them in* Multiplication, *and vary the tryal, as the nature of* Diviſion *does require, will eaſily underſtand it.*

CHAP. VI. *Of PROPORTION.*

IF of Four Numbers the firſt be to the ſecond, as the third to the fourth, thoſe numbers are called Proportional. The Ratio of one number to another is found by dividing the Antecedent by the Conſequent; as the Ratio of 31 to 7 is $4\frac{3}{7}$. *i. e.* quadruple ſuper-triparting ſevenths.

2. Wherefore if the ſame number multiply any two numbers, the Products will be proportional to the numbers multiplied; and if the ſame number divide two numbers, the Quotients will be proportional to the numbers divided.

$$\text{As } 4\times\left\{\begin{array}{l}7\ .\ 28.\\ 9\ .\ 36.\end{array}\right. \text{ and } 4\left)\begin{array}{l}28\ (7.\\ 36\ (9.\end{array}\right.$$

$$\text{Alſo } A\times\left\{\begin{array}{l}B\text{--}BA\\ C\ CA\end{array}\right. \text{ and } A\left)\begin{array}{l}BA.\ B\\ CA.\ C.\end{array}\right.$$

3. Wherefore if four numbers be proportional, the Product of the two Extremes will be equal to that of the two middle Quantities. e contra

$$\text{Now}\quad \begin{array}{l}7.9 :: 7\times4 . 9\times4 :: 28.36\\ 7\times9\times4 = 9\times7\times4.\end{array}$$

4. Hence follows the *Golden Rule of Proportion.* If of three numbers given you divide

the Rectangle made by the ſecond and third by the firſt, the Quotient will be the fourth Proportional. Let there be given 7. 9. 28. and call the fourth ſought, Q. Then

$$7.9 :: 28. Q.$$

$$7Q = 9\times 28 \text{ therefore } \frac{9\times 28}{7} = Q.$$

$$\text{Alſo } 5.\ 12 :: 8.\ \frac{8\times 12}{5}, \ i.\ e.\ 19\frac{1.}{5}$$

5. Of three given Numbers to find a fourth Proportional, the two firſt give you the Ratio: The Queſtion is concerning the third. In *Direct Proportion* the firſt Term is of the ſame kind with that by which the Queſtion is made. But in *Reciprocal Proportion*, the firſt Term is that by which the Queſtion is made.

6. *Direct Proportion* is when the greater that Term, by which which the Queſtion is made is, the greater the fourth will be alſo, and the leſſer it is, the leſſer the fourth.

7. *Reciprocal Proportion* is, when the greater that Term, by which the Queſtion is made, is, the leſſer the fourth will be; and the leſſer that Term is, the greater the fourth.

8. *Proportion* is Continual ∺ is, when all the Terms between the firſt and the laſt are both Antecedents, and conſequents in the ſame Proportion. 8. 12.

8.12.18. 27. are ∺ for 8. 12 :: 12. 18 :: 18. 27.

Alſo $\alpha.\ \beta.\ \frac{\beta\beta}{\alpha}\ \frac{\beta c}{\alpha q}\ \frac{\beta qq}{\alpha c}\ \frac{\beta qc}{\alpha qq}$ *&c.* ∺

Wherefore in this Series, if the laſt Term be ω, and the ſum of all the Terms be Z, then $Z\omega-$ will be the ſum of all the Antecedents, and $Z-\alpha$ the ſum of all the Conſequents.

9. If Four Quantities be proportional. $A\,\alpha :: B\,\beta$. they will alſo be Alternately Inverſly; and in Compoſition, Diviſion, and Mixtly proportional.

$A.\,\alpha :: B.\,\beta.$

Alternately $A.\ B :: \alpha\,\beta.$

Inverſly. $\alpha.\ A :: \beta\,B.$

In Compoſit. $A+\alpha.\,\alpha :: B+\beta.\,\beta.$

or $A+B.\ B :: \alpha+\beta.\,\beta.$

In Diviſion. $A-\alpha.\,\alpha :: B-\beta.\,\beta.$

or $A-B.\ B :: \alpha-\beta.\,\beta.$

Converſly. $A.\ A\pm\alpha :: B.\ B\pm\beta.$

or $A.\ A\pm B :: \alpha.\ \alpha\pm\beta.$

Mixtly. $A+\alpha.\ A-\alpha :: B+\beta.\ B-\beta.$

or $A+B.\ A-B :: \alpha+\beta.\ \alpha-\beta.$

10. In any number of Proportionals. As one of the Antecedents is to its Conſequent, ſo will be the ſum of the Antecedents to the ſum of the Conſequents. Let there

$A.\alpha :: B.\beta :: C.\gamma :: D.\delta$: Then will be
$A : \alpha :: A+B+C+D.\alpha+\beta+\gamma+\delta$.

$$\text{For}\left\{\begin{array}{l} A.\alpha :: B.\beta.\ \text{and in Compoſition.} \\ A+B.\ \alpha+\beta :: (B.\beta.)\ C.\gamma.\ \text{and} \\ A+B+C.\ \alpha+\beta+\gamma :: (C.\gamma.)\ D\,\delta. \end{array}\right.$$

Alſo in ∺ $\alpha.\beta :: Z-\omega.\ Z-x$. Wherefore
$\alpha Z-\alpha q = \beta Z-\beta\omega$ or $\beta Z-\alpha Z = \beta\omega-\alpha q$.

From hence $\left\{\dfrac{\beta\omega-\alpha q}{\beta-\alpha} = Z\right\}$ The Sum of the Terms $=$.

11. If the Antecedents of ſeveral Proportions be equal. As one of the Antecedents is to the ſum of its Conſequents, ſo the other to the ſum of its.

Say $A.B :: \alpha.\beta$. and $A.C :: \alpha.\gamma$. and $A.D :: \alpha.\delta$. then is $A.B+C+D :: \alpha.\beta+\gamma+\delta$. It appears from the foregoing Demonſtration; the Terms being placed Alternately.

12. If of two Ratio's the Conſequents are equal, they are to one another as the Antecedents? If the Antecedents are equal, they are Reciprocally as their Conſequents.

$$\tfrac{7}{1}.\tfrac{9}{1} :: 7.9.\ \text{and}\ \tfrac{1}{9}.\tfrac{1}{7} :: 7.\ 9.$$

13. If twice Four Quantities are alike Proportional, then their Sums, and their Differences are proportional.

14. If four Proportionals be either multiplied or divided by 4 other Proportionals,

the Products and Quotients will be proportional alſo. It follows from the third.

15. The Ratio of the Antecedent to the Conſequent, is compounded of the Ratio of the Antecedent to a third, and of that third to the Conſequent; or of the Ratio of the third to the Conſequent, and the Ratio of the Antecedent to hat third.

$$7.9 :: \times \begin{cases} 7.\ A. \\ A.\ 9. \end{cases} \text{ and } 7.9 :: \times \begin{cases} A.\ 9. \\ 7.\ A. \end{cases}$$

16. As for the Invention of the fourth Proportional in Aſtronomick Computations. If 100000 be the firſt Term, the fourth is found by the fifth. Cap. 4. Caſe 3. As 100000. 809202 :: 39875. 32260.

If 100000 be the ſecond or third Term, the fourth is found by 8. *Cap.* 5. 137658. 1001000 :: 126225. 91706.

17. As for the finding out of a Proportional Part from the given Difference of 2 numbers in the Canon of the *Proſthaphæreſes.* In the *Prutenick* Tables, at *Gr.* 62. Of the Anomaly of the firſt Epicycle of the Moon, the *Proſtaph.* is Gr. $4|\underline{1786,}$ Ablative, and the difference Gr. $0|\underline{0433.}$ How great a part is due to the Anomaly *Gr,* $62|\underline{5667}$? Say 1. $0|\underline{0433 ::}$ $0|\underline{5667}$. $0|\underline{245}$. *per* cap. 4. Sect. 5. Caſe 2. Then $4|\underline{1786+}0|\underline{0245=4}|\underline{2031}$. Which is the correct *Proſthaphæreſis.*

And on the other hand; if there be ſought the Anomaly of the firſt Epicycle of the Moon, congruous to the *Proſtaph.* of Gr. 4|2031, the next leſs in the Canon is Gr. 4|1786, anſwering to the Anomaly of Gr. 62. And the difference Gr. 0|0433. But 4|2031 − 4|1786 = 0|0245. Say therefore 0|0433. 0|0245 : : 1. 0|566+. The parts to be added to Gr. 62. And the ſought Anomaly is Gr. 62|566.

18. The Converſion of Sexageſimals into Decimals. And of Decimals into Sexageſimals is thus: 45 Sexageſimals are turned into Decimals, by dividing by 60, and 0|75 Decimals are turned into Sexageſimals, by multiplying by 60. As

60. 45′ : : 1. 0|75, } For
and 1. 0|75 : : 60. 45′. }

Diviſion by 60. Sets the ſeparating Line one ſtep toward the Left Hand, and divides by 6; and Multiplication by 60, ſets the ſeparating Line one ſtep toward the Right Hand, and multiplies by 6: Which Rule is obſervable.

But if there be ſeveral Sexageſimal Species joined to Integers, ſuppoſe 127. 32′. 00″. 09‴. 45⁗; uſe this Abridgment, under the Integers 127; ſet the Sexageſimals in an oblique Deſcent, and then, from the loweſt, divide each by 6; and ſet the Quotients in the next upper rank, till you come to the Integers.

127|5333784722
'32|002708333
''00|1625
'''09|75
iv 45

On the other hand. If Decimals be given, ſuppoſe 127|5333784722; multiply them continually by 6, and write the Products under one another, dropping one place toward the Right Hand, that the deſcent may be oblique. Obſerve the Example.

The degrees of the Equinoctial, with decimal Parts, ſuppoſe 236|4276 are turned into Decimal Parts of the Day, by dividing by 360, *i. e.* 6× 60.

And the Decimal Parts of the Day ſuppoſe

6) 236|4276
60) 39|4046×6
0|6567433 : ×60.

6567433 into Degrees, by multiplying by 360; *i. e.* 60×6. Obſerve the Example.

The degrees of the Equinoctial with Decimal Parts. Suppoſe 236|4276 are turned into hours, by dividing by 15; *i. e.* 3×5.

(3) 236|4276
5) 78|8092×3
15|76184×5

And hours with Decimal Parts are turned into Degrees, by multiplying by 15; *i. e.* 5×3.

Hours with decimal Parts. Suppoſe 15|76184 are turned into decimal Parts of the day, by dividing by 24; *i. e.* 4×6.

And decimal Parts of the Day. Suppoſe 6567433+ are turned into hours by dividing by 24, *i. e.* 6×4.

4) 15|76184
6) 3|94046×4
0|6567433×6

A Collect Sum, ſuppoſe 191374 is turned into an Expanſe Sum, by dividing continually by 60: and an Expanſe into a into a Collect, by multiplying it continually by 60.

19137|4
60) 318|9|3
53|0

So that if the Collected Sum be of Unities, *viz.* 191374°, the Expanded will be $53''.\ 09''\ 34^{\circ}$. *i. e.* 53 ſecond Sexagenes; *9* firſt Sexagenes; and 34 Unites. But if the Collected Sum be of the ſecond Sexageſimals, *viz.* $191370''$, the Expanded will be $53^{\circ}.\ 09'.\ 34''$.

19. Arithmetical Progreſſion is an equality of Differences. As 74. 12. 9. are in Arithmetical Proportion $7.\ 7-1 : 12.\ 12-13$.

20. Wherefore of four Arithmetically proportionals the Sum of the two Extremes is equal to the Sum of the two in the middle. $7+12-3=7-3+12$.

21. Add the ſecond to the 3, and ſubduct the firſt; the remainder will be the fourth. As $12+4-7=9$.

22. 'Tis called Continual Arithmetical Progreſſion, when every Term has the ſame increaſe; as 4. 7. 10. 13. 16. 19. the common increaſe is 3. So that every Term conſiſts of the firſt, and as many common Differences as it is ſteps diſtant from the firſt.

The Thirteenth Term will be $4+12\times3$, *i. e.* $4+36=40$.

23. Add the firſt Term to the laſt, and multiply them by the number of Terms, the product will be twice the ſum of the Progreſſion. $40+3\times13=572=2Z$, or twice the ſum of the Terms.

24. If over a Series of Terms, in Geometrical Progreſſion, you ſet ſo many Terms in Arithmetical Progreſſion? To any four Terms, in Arithmetical Proportion, there will anſwer four in Geometrical Proportion.

Index's, 6. 8. 10. 12. 14. 16. 18. 20.
Terms, 5.15.45.135.405.1215.3645.10935.

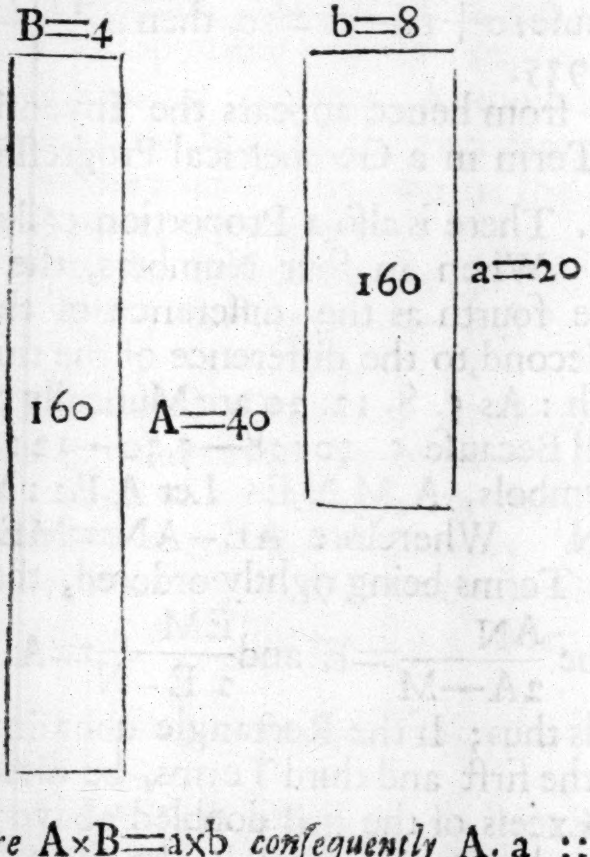
Becauſe $10+16-6=20$. then is $\frac{45\times1215}{5}=$ $=10935$.
And from hence appears the Invention of any Term in a Geometrical Progreſſion.

25. There is alſo a Proportion called Muſical. When in four Numbers, the firſt is to the fourth as the difference of the firſt and ſecond, to the difference of the third and fourth: As 5. 8. 12. 30 are Muſically proportional Becauſe $5.30::8-5.30-12::3.18$ In Symbols, A,M,N,E; Let $A.E::M-A.E-N$. Wherefore $AE-AN=ME-AE$ theſe Terms being rightly ordered, the Rule will be $\frac{AN}{2A-M}=E$ and $\frac{EM}{2E-N}=A$. In words thus; If the Rectangle contained under the firſt and third Terms, be divided by the Exceſs of the firſt doubled above the ſecond, the Quotient will be the fourth Term in Muſical Proportion: Wherefore the firſt Term doubled ought always to exceed the ſecond.

NOTES.

9. *If any have a mind to ſee the derivation of theſe Proportions, they may find them in Dr.* Wallis's *Treatiſe of* Algebra. *Cap.* 19. *They are too much to inſert here.*

7. Reciprocal Proportion *may be beſt explained by* 2 *equal Rectangles.*



Here A×B=axb *conſequently* A. a :: b. B. *Theſe Quantities are ſaid to be Reciprocally proportional, becauſe* A *is to* a, *not as* B *is to* b *but backwards,* A. a :: b. B.

17. *In the* Prutenick *Tables,* &c. [*Theſe Examples will ſtand thus.*

```
0|5667
3340|0
00000
 0000
  227
   17
    1
------
0|0245
```

```
433)2450 (0|565+
    2165
    ----
     2850
     2598
     ----
     2320
     2165, &c.
```

18. *The converſion of Sexageſimals into Decimals ,* &c. *Divide an Unite into* 60 *parts , and the ſame into* 10 *; there muſt of neceſſity be contained in every tenth* 6 *ſixtieths; and conſequently the ſixtieths muſt be divided by* 6, *to reduce them into tenths ; and tenths multiplied by* 6, *to reduce them into ſixtieths. And this is the Foundation of all that belongs to this Matter ; and will eaſily ſatisfie any one concerning this, or the following Operation.*

19. Degrees of the Equinoctial with Decimal Parts, *&c.*] *The Reaſon is this. The whole Circle conſiſts of* 360 *degrees, that is,* 36x10, *or ten times* 36 *degrees; and therefore every tenth of the whole Circle contains* 36 *degrees: So that you muſt divide Equinoctial Degrees by* 36, *to caſt them into tenths, and multiply tenths by* 36, *to caſt them into* 360*ths*.

CHAP. VII.

Concerning the greateſt Common MEASVRE by which given Numbers are reduced to the leaſt Terms.

1. THE greateſt common Diviſor or Meaſure of two Numbers is found by a perpetual Diviſion of the greater by the leſſer, and of the Diviſor by the Remainder. For that Diviſor, which firſt divides its Dividend without any Remainder, will be the common Meaſure of the given Numbers. As of the number 899, and 744 the greateſt common Meaſure will be 31. As thus,

```
744) 899 (1
     744
     ---
     155|744 (4
        |620
        -----
         124|155 (1
            |124
            -----
              31|124 (4
                |124
                -----
                 000
```

$$\begin{array}{cccccc}
 & & 31 & 124 & 155 & \\
31) & \cancel{1}\cancel{2}\cancel{4}) & \cancel{1}\cancel{5}\cancel{5}) & \cancel{7}\cancel{4}\cancel{4}) & \cancel{8}\cancel{9}\cancel{9} & (\cancel{1}\cancel{4}\cancel{1}\cancel{4} \\
 & \cancel{1}\cancel{2}\cancel{4} & \cancel{1}\cancel{2}\cancel{4} & \cancel{6}\cancel{2}\cancel{0} & \cancel{7}\cancel{4}\cancel{4} &
\end{array}$$

2. Numbers are reduced to the leaſt Terms of the ſame Ratio by a diviſion of them both by their greateſt Common Meaſure. As 899 and 744 are reduced to 29 and 24, the leaſt Terms in that Ratio, both being divided by 31, their greateſt Common Meaſure.

So $\frac{3Aq}{6A}$ are reduced to $\frac{A}{2}$, by diviſion of them both by 3A. And $\frac{4Acc}{6Aqq}$ are reduced to $\underline{2Aq}$, by a diviſion of them both by 2Aqq.

Alſo $\frac{\overset{3}{B}A}{B}$ is reduced to A, by a Diviſion of them both by B. For that which is done by Multiplication, is undone by Diviſion.

3. Wherefore, If the greateſt Common Meaſure of two Numbers be 1; Theſe 2 Numbers are called Prime Numbers, and are the leaſt which bear that Proportion to one another. Such are 29 and 24.

4. If a Number be a Prime Number, with reſpect to two other, 'twill alſo be a Prime Number to the Product of thoſe two multiplied into one another? Hence the manag-

ing proportional Numbers may be made more eaſie oftentimes. For Example,

$$\overset{\overset{1}{\cancel{3}}}{1\cancel{2}}.\quad \overset{2.}{\cancel{8}} :: \overset{5}{1\cancel{5}}.\quad 10$$

5. You muſt remember always when a Fraction is before you, to reduce it to the eaſt Terms. As $\frac{344}{899}$ to $\frac{24}{29}$

NOTES.

1. *Concerning this matter you are to conſult the ſecond Propoſition of the ſeventh Book of* Euclides Elements.

4. *In this Example* 12 8 : : 15. 10. *Firſt you (as is before directed) reduce the Ratio of* $\frac{32}{8}$ *to* $\frac{3}{2}$, *by dividing them by their common Meaſure* 4. *ſo that* 3. 2 : : 15. 10. *Then dividing* 3 *and* 15 *by their common Meaſure* 3. *The Figures will be theſe*; 1 2 : : 5. 10.

CHAP.

CHAP. VIII.

Concerning FRACTIONS.

1. UNity, or any other *Totum* or whole thing, may be conceived diviſible into never ſo many parts; which receive their Name from the number of parts into which *Unity* is ſuppoſed to be divided. As, if *Unity* be ſuppoſed to be divided into two equal parts, they are called ſeconds; if into three, thirds, *&c.*

2. Fractions conſiſt of two Terms one above the other, with a Line drawn between them; the lower of which tells you, that *Unity* is to be ſuppoſed to be divided into juſt ſo many equal parts, called the Denonator. The upper tells you, how many of thoſe parts belong to that Fraction; and is called the Numerator. As 4 Numerator,
5 Denominator.
This Fraction does denote four fifth parts of an Integer; that is: Suppoſe an Integer divided into five parts, this denotes four of thoſe parts.

3. Therefore whatſoever Ratio the Numerator has to the Denominator, the ſame has the Fraction to *Unity*.

$4 . 5 :: \frac{4}{5} . 1 .$ $R . S :: \frac{R}{S} . 1$

4. And becauſe a Ratio may be expreſſed innumerable ways, and that in higher and higher Terms, infinitely in the ſame habitude one to another; it follows, that Fractions may be expreſſed innumerable ways alſo; as $\frac{5}{12}$ (the leaſt Terms in that Ratio, and in Latin called *Quincunx*) is equivalent to $\frac{10}{24}$, $\frac{20}{48}$, $\frac{25}{60}$, $\frac{45}{108}$, and any other numbers that ariſe from the multiplication of 5 and 12 into any other number, by Sect. 2. Chap. 6.

5. Wherefore the Terms of equal Fractions are proportional; and if the Terms are proportional, the Fractions are of equal value.

6. If the Numerator be leſs than the Denominator, the Fraction is leſs than an *Unite*; if equal, it ſignifies an *Unite*? If the numerator be greater than the Denominator, the Fraction is as much greater than an *Unite* as that is greater than this? A Fraction is reduced to *Unites* by dividing the Numerator by the Denominator.

as $\frac{31}{7} = 4\frac{3}{7}$ and $\frac{CR+SA}{R} = C+\frac{SA}{R}$

On the contrary Unites are reſolved into Fractions, by multiplying them by the Denominator of the Fraction, as $1=\frac{2}{2}$ or $\frac{3}{3}$, &c. and $4\frac{3}{7}=\frac{28+3}{7}=\frac{31}{7}$, and $C+\frac{SA}{R}=\frac{CR+SA}{R}$.

CHAP. IX.

Concerning the Addition *and* Subduction *of* Fractions.

1. IF the Fractions to be added be of divers Denominations. Firſt, They are to be reduced to the ſame Denomination by dividing the Denominators by their greateſt common Meaſure, and multiplying the Terms by each others Quotients; then the Numerators of the ſo found Fractions are to be added or ſubducted. And laſtly, write that common Denominator under the Sum or Difference.

2. And if Integers be mixed with the Fractions, they are to be numbred apart:

For Example, If you would take out of $6\frac{1}{18}$ theſe Fractions, $\frac{13}{16}$ and $2\frac{7}{12}$, firſt of all you muſt add $\frac{13}{16}$ and $2\frac{7}{12}$ and they will come to 2 and $\frac{39+28}{48}$ or $\frac{67}{49}$, viz. $3\frac{19}{48}$ which being taken out of $6\frac{1}{18}$ there will remain $2\frac{95}{144}$ as in the Example.

$$\begin{array}{c} 67 \\ 39+28 \\ 4)\ \frac{\cancel{1}\cancel{3}}{\cancel{1}\cancel{6}}\ 2\ \frac{\cancel{7}}{\cancel{1}\cancel{2}} \text{ is} \\ \underbrace{2 \qquad\quad 3}_{48} \end{array} \qquad \begin{array}{c} 57 \qquad\qquad 8 \\ 3\frac{\cancel{1}\cancel{9}}{48}\ \text{out of}\ 6\frac{1}{18} \\ 6)\ \underbrace{8 \qquad\qquad 3}_{144} \end{array} \qquad \begin{array}{c} 8 \text{ out of } 5\frac{152}{144} = 6\frac{8}{144} \\ \text{take} \\ 3\frac{57}{144} \\ \text{there remains} \\ 2\frac{95}{144} \end{array}$$

Add $\frac{A}{B}$ and Z, the Sum will be $\frac{A+ZB}{BA}$

Out of $\frac{A}{B}$ take $\frac{B}{C}$ there remains $\frac{CA-Bq}{BC}$

$$\begin{array}{c} BE+DA \\ B+D \\ \overline{}\qquad\overline{} \\ C)CA \times CE \\ A \qquad\quad E \\ \underbrace{\qquad\qquad\qquad}_{CAE} \end{array}$$

CHAP. X.

Concerning the Multiplication *and* Diviſion *of* Fractions.

1. MUltiplication compares the Numerator of one and the Denominator of the other, that is, reduces them to their leaſt Terms, and multiplies the Terms of the ſame name.

2. *Diviſion* compares the Terms of the ſame Name; and multiplies the Terms of a different Name.

3. If Integers be mixed with Fractions, they are to be reſolved into a Fraction.

Examples of Multiplication.

$\dfrac{\overset{1}{\cancel{9}}}{\underset{4}{\cancel{16}}}$ upon $\dfrac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{27}}}$ is $\dfrac{5}{12}$ | $\dfrac{\overset{4}{\cancel{8}}}{9}$ upon $\dfrac{5}{\underset{3}{\cancel{6}}}$ is $\dfrac{20}{27}$ |

$\dfrac{5}{1.}$ upon $\overset{13}{3\dfrac{\cancel{1}}{4}}$ is $\dfrac{65}{4}$ (16$\dfrac{1}{4}$

$$\frac{A}{B}\ \text{upon}\ B = \underset{\cdot}{A}\ \Big|\ \frac{A}{B}\ \text{upon}\ Z = \frac{ZA}{B}\ \Big|$$

$$\frac{A}{B}\ \text{upon}\ 3\,\frac{ZA}{C} = 3\,\frac{ZAq}{BC}.$$

Examples of Diviſion.

$$\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{16}}}\Bigg)\frac{\overset{5}{\cancel{15}}}{\underset{7}{\cancel{28}}}\Bigg(\frac{20}{21}\ \Bigg|\ \frac{8}{\underset{3}{\cancel{15}}}\Bigg)7\frac{\overset{37}{\cancel{2}}}{\underset{1}{\cancel{5}}}\Bigg(\frac{111}{8}\Bigg(13\frac{7}{8}\ \Bigg|$$

$$\frac{\overset{1}{\cancel{3}}}{4}\Bigg)\frac{\overset{3}{\cancel{9}}}{1}\Bigg(\frac{12}{1}$$

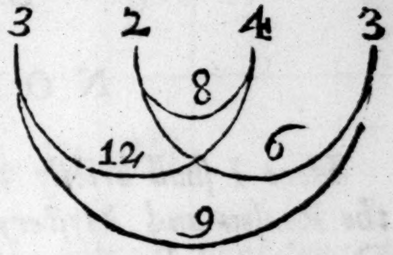
$$\frac{D}{1}\Bigg)\frac{Aq}{B}\Bigg(\frac{Aq}{DB}\ \Bigg|\ \frac{A}{D}\Bigg)\frac{BC}{1}\Bigg(\frac{BCD}{A}\ \Bigg|\ \frac{A}{B}\Bigg)\frac{BC}{1}\Bigg(\frac{BqC}{A}$$

$$\frac{B.}{A\cdot}\Bigg)\frac{BC}{1}\Bigg(\frac{CA}{1}\ \Bigg|\ \frac{A\ c}{C}\Bigg)\frac{B\ c}{D}\Bigg(\frac{BcC}{DAc}$$

4. What Number is $\frac{2}{7}$ of 21? Multiply 21 by $\frac{2}{7}$ for 1 $\frac{2}{7}$ :: 21 . 6. or 7 . 2 :: 21 . 6.

5 Of what Number does 6 contain $\frac{2}{7}$? Divide 6 by $\frac{2}{7}$. for $\frac{2}{7}$ 1 :: 6 . 21. or 2 . 7 :: 6 . 21.

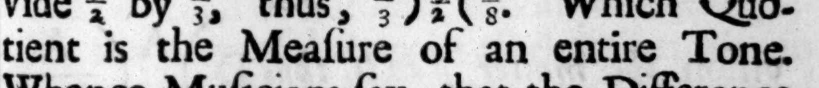
6. The



6. The antient Writers of Muſick are wont to connect the terms of Ratio's, either to be continued or diminiſh'd, with Curve Lines after this manner: If the Ratio's be 3 to 4 and 4 to 3.

7. To continue Ratio's is to multiply them as if they were Fractions. To ſay let theſe Ratio's 3 to 2 and 4 to 3 be continued, is as much as to ſay, let $\frac{3}{2}$ be multiplied upon $\frac{4}{3}$, and ſo there will ariſe $\frac{12}{6}$. Now the Ratio of 12 to 6, is double. Wherefore a twice-and-an-half-Ratio upon a once-and-one-third-Ratio produceth a double Ratio, *that is*, as Muſicians ſpeak, a *Diapaſon* is compounded of a *Diapente* and a *Diateſſaron*.

Imminution of Ratio's is done by Diviſion, as to ſay, take out of the Ratio of 3 to 2, the Ratio of 4 to 3, is as much as to ſay, divide $\frac{3}{2}$ by $\frac{4}{3}$, thus, $\frac{4}{3})\frac{3}{2}(\frac{9}{8}$. Which Quotient is the Meaſure of an entire Tone. Whence Muſicians ſay, that the Difference between a *Diapente* and a *Diateſſaron* is a Tone. As in this Line or Chord divided into 12 parts.



NOTES.

Here I ſhall briefly and ſymbolically delive the Reaſon and Myſtery of Multiplication *and* Diviſion *of Fractions. To that purpoſe I wil make uſe of this Example, Suppoſe that you ar to multiply* $\frac{A}{B}$ *upon* $\frac{C}{D}$, *'Tis plain, firſt tha* $A\times\frac{C}{D}=\frac{AC}{D}$; *becauſe you are not only to multiply* A *upon* $\frac{C}{D}$ *but* $\frac{A}{B}$; *there that Quantit which ariſes from a Multiplication of* A *upon* $\frac{C}{D}$ $=\frac{AC}{D}$ *muſt be divided* B, *that is,* B *muſt b multiplied upon* D *the Denominator of the Fraction, and then it will be* $\frac{AA}{BD}$. *As for* Diviſion *the matter ſtands thus* $D\Big)\frac{A}{B}\Big(\frac{A}{BD}$ *and* $\frac{A}{BD}\times C=\frac{AC}{BD}$; *therefore* $\frac{D}{C}\Big)\frac{A}{B}\Big(\frac{AC}{BD}$

CHAP

CHAP. XI.

Some eaſie Examples, which prepare the Way for Analyticks.

1. OBſerve firſt, that for brevity ſake and to help the Fancy, we make uſe of theſe Symbols, A and E ſignifie two Magnitudes of which A is the greater, E the leſſer, Æ their Rectangle, Z their Summ, X their Difference, .Z the Summ of their Squares, X the Difference of their Squares, 'Z the Summ of their Cubes, 'X the Difference of their Cubes; A, M, E ſtand for three quantities in continual Proportion; A, M, N, E for Four; Q: C: QQ. QC: ſignifie ſuch and Powers; √ denotes the Root or Side of a ſimple Power. But if the Power be included between two Points at both ends, it ſignifies the univerſal Root of all that Quantity ſo included; which is ſometimes alſo ſignified by *b* and *r*, as the √*b* is the Binomial Root, the √*r* the Reſidual Root (or of the Apotome.) ═ is a note of Equality.

2. There are two Numbers of which A is the greater, E the leſſer. I ask what is their Summ? what is their Difference? what

is their Rectangle? what is the Summ of their Squares? what is the Difference of their Squares? what is the Summ of their Summ and Difference? what is the Difference of their Summ and Difference? what is the Rectangle of their Summ and Difference? what is the Square of the Summ? what is the Square of the Difference? what is the Summ of the Squares of the Summ and Difference? what is the Difference of the Squares of the Summ and Difference? what is the Square of the Rectangle?

$$Z=A+E \qquad X=A-E$$
$$\mathcal{Z}=Aq+Eq \qquad \mathcal{X}=Aq-Eq$$
$$Z+X=2A \qquad Z-X=2E$$
$$\tfrac{1}{2}Z+\tfrac{1}{2}X=A \qquad \tfrac{1}{2}Z-\tfrac{1}{2}X=E$$
$$ZX=Aq-Eq=\mathcal{X} \qquad Zq\,.\,\mathcal{X} :: Z\,.\,X$$
$$Zq=Aq+2AE+Eq=\mathcal{Z}+2\text{Æ}.$$
$$Xq=Aq-2AE+Eq=\mathcal{Z}-2\text{Æ}.$$
$$Zq+Xq=2Aq+2Eq=2\mathcal{Z}.$$
$$Zq-Xq=4AE \qquad \tfrac{1}{4}Zq-\tfrac{1}{4}Xq=\text{Æ}$$
$$\text{Æ}q=AqEq.$$

3. There are two Quantities, the Summ of which is Z and the greater A. What is the lesser? what is their Difference? what their Rectangle? what the Summ of their Squares? what the Difference of their Squares.

$E = Z - A.$ $X = 2A - Z.$ $\text{Æ} = ZA - Aq$

$Z = Zq - 2ZA + 2Aq.$ $X = 2ZA - Zq$

But if the leſſer E be given,

$A = Z - E.$ $X = Z - 2E.$ $\text{Æ} = ZE - Eq.$

$Z = Zq - 2ZE + 2Eq.$ $X = Zq - 2ZE$

4. There are two Quantities, of which the difference is X, and the greater A; What is the leſſer? what is their Sum? what their Rectangle? what the Sum of their Squares? what the difference of their Squares?

$E = A - X$ $Z = 2A - X$ $\text{Æ} = Aq - XA$

$Z = 2Aq - 2XA + Xq$ $X = 2XA - Xq$

But if the leſſer E be given,

$A = E + X$ $Z = 2E + X$ $\text{Æ} = Eq + XE$

$Z = 2Eq + 2XE + Xq$ $X = 2XE + Xq$

5. There are two quantities of which the greater has the ſame proportion to the leſſer, as R to S; and the greater of them is ſuppoſed to be A: What is the leſſer? what is their Sum? what is their Difference? what their Rectangle? what the Sum of their Squares? what the difference of their Squares?

$E = \frac{SA.}{R}$ $Z = \frac{RA + SA}{R}$ $X = \frac{RA - SA}{R}$

$\text{Æ} = \frac{SAq}{R}$ $Z = \frac{RqAq + SqAq.}{Rq}$

$X = \frac{RqAq - SqAq.}{Rq}$

But if the leſſer E be given,

$$A=\frac{RE}{S} \quad Z=\frac{RE+SE.}{S} \quad X=\frac{RE-SE}{S}$$

$$\AE=\frac{REq}{S} \quad Z=\frac{RqEq+SqEq.}{Sq}$$

$$X=\frac{RqEq-SqEq.}{Sq}$$

6. There are two Quantities, of which the Rectangle is Æ; and the greater A: What is the leſſer? what is their Sum? what is their Difference? what is the Sum of their Squares? what is the Difference of their Squares?

$$E=\frac{\AE.}{A} \quad Z=\frac{Aq+\AE.}{A} \quad X=\frac{Aq-\AE}{A}$$

$$Z=\frac{Aqq+\AE q}{Aq} \quad X=\frac{Aqq-\AE q}{Aq}$$

But if the leſſer E be given,

$$A=\frac{\AE.}{E} \quad Z=\frac{\AE+Eq.}{E} \quad X=\frac{\AE-Eq}{E}$$

$$Z=\frac{\AE q+Eqq}{Eq} \quad X=\frac{\AE q-Eqq}{Eq.}$$

7. An

7. And from hence many equalities ariſe. Let us take the Sum and Difference for Examples.

$$Z = A + E = 2A - X = 2E + X = \frac{Aq + Æ}{A} = \frac{Æ + Eq}{E}, \&c.$$

$$X = A - E = 2A - Z = Z - 2E = \frac{Aq - Æ}{A} = \frac{Æ - Eq}{E}, \&c.$$

After this manner the ſame quantity will admit of divers Interpretations and Diverſities.

NOTES.

Here (no more being neceſſary) I ſhall only adviſe young Students of this ſort of Learning, *to practiſe theſe Queſtions ſeveral times over, and to draw Figures for ſome of the Equations; whereby they may gain a full underſtanding of the matter.*

CHAP. XII.

Of the Geneſis and Analyſis of POWERS.

1. BEcauſe all things are to be reſolved into thoſe parts of which they conſiſt; firſt we muſt know of what parts every Power conſiſteth. Now Powers ariſe from the ſide or Root multiplied once or more times into it ſelf. The Root multiplyed upon it ſelf makes a Square. The Square, multiplied upon the Root, makes a Cube. A Cube upon its Root makes a Quadrato-Quadrate, or fourth Power [4]. This again, upon its Root, makes a Quadrato-Cube, or fifth [5] And after this manner there are made a ſixth [6], a ſeventh [7], an eighth [8], a ninth [9], a tenth [10] Power, *&c.*

2. Wherefore the Procreation of Powers from a ſingle Root, which conſiſts of one Figure or Note, hath no difficulty at all.

The

The former Table of POWERS *from a ſingle Root.*

Roots.	2	3	4	5	6	7	8
	q	c	qq	qc	cc	qqc	qcc
1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256
3	9	27	81	243	729	2187	6561
4	16	64	256	1024	4096	16384	65536
5	25	125	625	3125	15625	78125	390625
6	36	216	1296	7776	46656	279936	1679616
7	49	343	2401	16807	117649	823543	5764801
8	64	512	4096	32768	262144	2097152	16777216
9	81	729	6561	59049	531441	4782969	43046721

3 But Powers from a Binomial Root are generated after this manner.

The Geeneſis of Powers *from a Binomial Root.*

$$\begin{array}{l}
\quad A+E \\
\quad A+E \\
\hline
Aq+AE \\
\quad +AE+Eq \\
\hline
Aq+2AE+Eq.\quad \text{Quadrat.} \\
\qquad A+E \\
\hline
Ac+2AqE+AEq \\
\quad +AqE+2AEq+Ec \\
\hline
Ac+3AqE+3AEq+Ec.\quad \text{Cube.} \\
\qquad A+E \\
\hline
Aqq+3AcE+3AqEq+AEc \\
\qquad +AcE+3AqEq+3AEc+Eqq \\
\hline
Aqq+4AcE+6AqEq+4AEc+Eqq.\ \text{Quadrato-quadrat.} \\
\qquad A+E\ \text{\&c.}
\end{array}$$

4. And by this Artifice may be framed a Table of Powers aſcending in the ſcale from a Binomial Root; which I call the LATTER TABLE.

Side or Number	2	3	4	5	6	7	8	9	10
A	Aq	Ac	Aqq	Aqc	Acc	Aqqc	Aqcc	Accc	Aqqce
E	2AE	3AqE	AcE	5AqqE	6AqcE	7AccE	8AqqcE	9AqccE	10AcccE
	Eq	3AEq	6AqEq	10AcEq	15AqqEq	21AqcEq	28AccEq	36AqqcEq	45AqccEq
		Ec	4AEc	10AqEc	20AcEc	35AqqEc	56AqcEc	84AccEc	120AqqcEc
			Eqq	5AEqq	15AqEqq	35AcEqq	70AqqEqq	126AqcEqq	210AccEqq
				Eqc	6AEqc	21AqEqc	56AcEqc	126AqqEqc	252AqcEqc
					Ecc	7AEcc	28AqEcc	84AcEcc	210AqqEcc
						Eqqc	8AEqqc	36AqEqqc	120AcEqqc
							Eqcc	9AEqcc	45AqEqcc
								Eccc	10AEccc
									Eqqcc

5. Every intermediate Species of each row is made up of the two Species of the former row next unto it on both ſides; namely, the Power of A in that above it, and of E in that below it. The Number prefixed to it is made up of the two Numbers next adjoyning in the foregoing Power, ſo that you may continue it as far as you pleaſe.

6. In this Table the two extreme Powers in each rank are called the Diagonal part of the Power; the intermediate Species, to which the Unciæ (or Figures) are prefixed, are called the Complements; all the Complements together with the Power of E are called the Gnomon.

7. From this Table 'tis manifeſt, that the Square from a Binomial Root conſiſts of the Diagonal Squares of the greater and the leſſer Nomen, with two Rectangles made by them. The Cube of the Diagonal Cubes of the greater and the leſſer, with three Solids made by the Square of the greater upon the leſſer, and three other Solids made by the greater upon the Square of the leſſer. Thus from this Table you may learn the Parts of which higher Powers conſiſt.

8. This Table (being full of fine Myſteries) do's alſo ſhew how many places belong to every Power or to its Complements.

For becauſe between two Squares there can be but one kind of Complement, therefore the Square requires but one place for its Complement. But becauſe between two Cubes there are two kinds of Complements, therefore the Cube does require two places for its Complements.

NOTES.

5 Every intermediate Species, *&c.*] *That is, (for inſtance take the fifth Power) the third Species* AcEq (*before which, in the fourth Power, ſtand next to it* AcE *and* AqEq) *takes from the former* Ac, *and from the latter* Eq. *And ſo of all other Species.*

But the Number prefixed, &c.] *To keep to the former inſtance,* 10 *prefixed to* AcEq *is the Aggregate of* 4 *and* 6 *prefixed to* AcE *and* AqEq *in the foregoing Power.*

CHAP. XIII.

Concerning the Geneſis or Compoſition of POWERS.

1. SUppoſe you are to compoſe a Square from the Root 57. Then the greater Note A is 5 or 50. The leſſer E is 7.

Let them be written at the diſtance of one Place or Degree from one another, and draw a Line under them. Under 5 ſet its Square 25, under 7 its, which is 49, then double 5 and multiply its double by 7, and the Product will be two Rectangles, *viz.* 70; and write 70 in the intermediate Place. Add them up as they ſtand, and the Product will be 3249 for the ſquare of the ſide 57.

25	7	Aq	
7	0	2AE	Gnomon.
	49	Eq	
32	49		

2. Suppoſe you were to make a Cube from the Root 57. Let them be ſet down two Places one from the other, and draw a Line under them. Under 5 ſet its Cube 125, and under 7 its Cube 343. Then treble the Square of 5, and multiply it ſo trebled by 7, and the Product will be 525; three greater Solids to be ſet in the former intermediate Place; alſo treble 5, and multiply it ſo trebled by 49 the Square of 7, the Product will be 735; three leſſer Solids to be ſet in the latter intermediate Place, that is, one Place further towards the right hand. Add them up

5	7		
125	Ac		
52	5	GAqE	Gnom.
7	35	3AEq	
	343	Ec	
185	193		

up as they ſtand, and the Product will be 185193 for the Cube of 57.

3. If the Root conſiſt of many Figures, as this 57209. Firſt you muſt compoſe the Power of the two firſt Figures 57; then taking 57 for A, and the next Figure 2 for E, proceed as before; and ſo till you have diſpatch'd all the Figures in the Root.

5	7	2	0	9	The Root.	
25		Aq				
7	0	2AE	} Gnomon.			
	49	Eq				
32	49		Aq			
	22	8	2AE	} Gnomon.		
		4	Eq			
32	71	84	00		Aq	
	1	02	96	0	2AE	} Gnom.
				81	Eq	
32	72	86	96	81	The Square.	

5

5	7	2	0	9		Root.
125					Ac	
52	5				3AqE	
7	35				3AEq	Gnomon.
	343				Ec	
185	193				Ac	
1	949	4			3AqE	
	6	84			3AEq	Gnomon.
		8			Ec	
187	149	248			Ac	
	88	339	680	0	3AqE	
		13	899	60	3AEq	Gnomon.
				729	Ec	
187	237	601	597	329.		Cube.

4. From what has been ſaid it will not be hard to compoſe any Power.

CHAP.

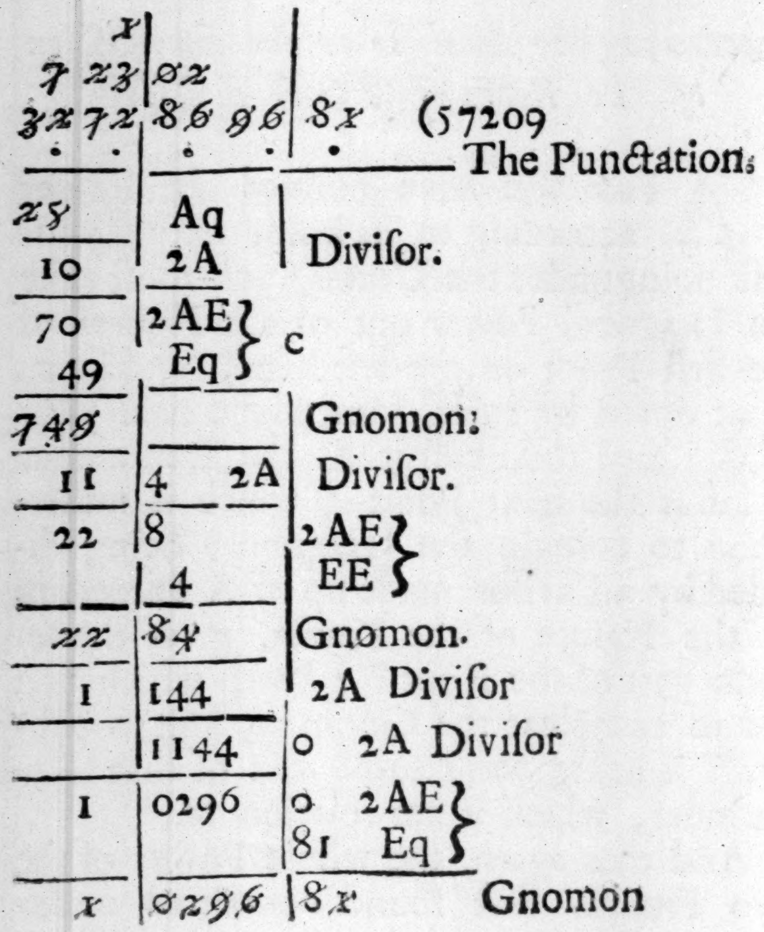
CHAP. XIV.

Concerning the Analyſis or the Extraction of the Roots of POWERS.

1. AFter you have pointed the Power according to its kind, ſetting your firſt point under the Unites, you muſt take the Diagonal Power out of the Figures of the firſt Point on the left hand, and ſet its Root down by itſelf, ſomewhere near your work; and then call it **A**. the Remainder as far as the next Point (which is underſtood to contain the Gnomon) being divided by a Diviſor made up of **A** according to the Nature of the Power, gives the ſecond part of the Root E; with which you are to compleat the Gnomon, and to ſubſtract it being compleated out of that Remainder, which it has relation to.

And thus again, the whole Power of the two Figures firſt found contained under the two firſt Points being ſubſtracted, there will remain a Gnomon reaching to the third Point; by means of which and your Diviſor, as before, you may find the third part of the Root; as before.

The Extraction of the Square Root.



The

The Extraction of the Cube Root.

~~2088~~					
~~62044~~		~~353~~			
~~187237~~		~~601~~	~~580~~	~~329~~	(~~57209~~
~~125~~		Ac			
7	5	3 Aq			
	15	3 A			
7	65	Diviſ.			
52	5	3 AqE			
7	15	3 AEq			
	343	Ec			
~~60~~	~~193~~	Gnom.			
	974	7	3 Aq		
	1	71	3 A		
	976	41	Diviſor		
1	949	4	3 AqE		
	6	84	3 AEq		
		8	Ec		
~~1~~	~~956~~	~~248~~	Gnom		
	98	155	2	3 Aq	
		17	16	3 A	
	98	172	36	Diviſor	
	9	815	520	0	3 Aq
			171	60	3 A
	9	815	691	60	Diviſor
	88	339	680	0	3 AqE
		13	899	60	3 AEq
				729	Ec
1	~~88~~	~~353~~	~~580~~	~~329~~	

5	7	2	0
25			
7	0		
	49		
32	49		
	22	8	
		84	
32	71	84	00

2. If the propoſed Number be not a true figurate Number, (that is, an exact Square) but ſomething remains when the Work is done, you may ſet after the remaining Figures as many Ciphers as you pleaſe, and continue your Extraction.

3. From what has been ſaid it will not be hard to extract the Root of any Power, ſo that you conſult the Table, to learn thence the conſtituent parts of each Power.

CHAP. XV.

Concerning Surd Roots.

1. IN a rank of Numbers in continual Proportion. As the firſt is to the laſt, ſo will be the Power of the firſt, whoſe Index is equal to the Number of terms, wanting one to the like Power of the ſecond. Let there be, . ∺ A, M, N, E,

$$\text{Becauſe}\left\{\begin{array}{l} A\ M :: A.\ M \\ M\ N :: A.\ M \\ N\ E :: A.\ M \end{array}\right\}\begin{array}{l}\text{there will be} \\ A.\ E :: Ac.\ Ac.\end{array}$$

2. Plane Numbers or ſolid Numbers are then called *like*, when their reſpective ſides are proportional.

3. Like plain Numbers are in a duplicate Ratio (that is as the Squares) of their correſponding ſides.

Therefore like plain Numbers are to one another, as a Square number is to a Square number. Alſo like ſolid Numbers are in an triplicate Ratio of their correſponding ſides, that is as their Cubes are to one another. Therefore like numbers ſolid bear a cubical Proportion to one another.

4. And generally all like Figurate Numbers of divers dimentions are in a Ratio of their correſponding ſides, equimultiplicate to the number of dimenſions of which they conſiſt. Let the dimenſions be 4, *viz.* ABCD of the one, and EFGH of the other in the Ratio of R to S.

$$\text{Becauſe}\left\{\begin{array}{l} A.E :: R.S \\ B.F :: R.S \\ C.G :: R.S \\ D.H :: R.S \end{array}\right\}\begin{array}{l}\text{By Multiplication} \\ \text{ABCD. EFGH ::} \\ \text{Rqq. Sqq.}\end{array}$$

5. If a number be not a true figurate Number of its kind, its Root is call'd a ſurd Root, and is ſo denoted. √q6. √q4. √qq20. √qc13. that is the ſide or Root of the Square 6, the ſide of the Cube 4, the ſide of the Quadrato-quadrate 20, the ſide of the Quadrato Cube 13, *&c.*

6. Thoſe ſurd Roots are call'd Commenſurable, whoſe numbers reduc'd to the leaſt Terms, become true figurate Numbers of their kind, and are therefore as one number to another. As √q12 and √q147, being reduced to their leaſt Terms by their greateſt Common Meaſure √q3, become √q4 and √q49 ; that is, 2 and 7.

Wherefore the √q12 being to √q147, as 2 to 7, they are commenſurable. So √c40 and √c1715 are to one another, as 2 to 7 ; becauſe being divided by their greateſt Common Meaſure √c5, they become √c8 and √c343, and therefore commenſurable.

7. Surd commenſurable Roots are added and Subſtracted, if the homogene Power of the ſum or difference of their like numbers found, (as before) be drawn upon their Common Meaſure. As √q147+√q12 is the √q243 ; that is, the Root of the Square of 7+2 (*viz.* 81) drawn upon √q3. And √q147- √q12 is √q75 ; that is, the Root of the Square of 7—2, (*Viz.* 25.) drawn upon √q3. Alſo √c1715+√c40 is √c3645 ; that is, the Root of the Cube of 7+2, (*viz.* 729) drawn upon the √c5, and √c1715—√c40 is √c625, that is, the Root of the Cube of 7—2 (*viz.* 125) drawn upon the √c5.

The Examples ſtand thus.

$\surd q3)\ \ \surd q147\ \ (\surd q49.7$

$\surd q12\ \ (\surd q4.2$

Sum $\surd q\,243$ $\surd q81.9$

Diff. $\surd q\,75$ $\surd q25.5$

$\surd c1715\ \ (\surd c343.7$

$\surd c5)\ \ \surd c\ 40\ \ (\surd c\ 8.2$

Sum $\surd c3645.$ $\surd c729.9$

Diff. $\surd c\ 625.$ $\surd c125.5$

$\surd 12+\surd\frac{27}{4}$

or

$\surd\frac{48}{4}+\surd\frac{27}{4}$

$\surd 3)\ \surd 48.\ (\surd 16.4$

$\surd 27.\ (\surd 9.3$

$\surd\frac{147}{4}\ \ \surd 49.7$

$\surd\frac{3}{4}\ \ \surd 1.1$

$\surd\frac{245}{12}+\surd\frac{5}{12}$

$\surd 5)\ \surd 245\ (\surd 49.7$

$\surd 5.\ (\surd 1.1$

$\surd\frac{320}{12}\ \ \surd 64.8$

$\surd\frac{180}{12}\ \ \surd 36.6$

8. But incommenſurable ſurd Roots and Heterogeneal are added and ſubſtracted by the Signs + or −, as $\surd q7+\surd q4$ and $\surd c10-\surd c5$.

9. If a figurate Number be multiplied by another figurate Number ~~of the~~ ſame kind, the product will be a figurate Number of the ſame kind, whoſe Root is equal to the product of the Roots of the multiplied Numbers.

And if a figurate Number be divided by another of the ſame kind, the quotient will be a figurate Number of that kind, whoſe Root is equal to the quotient of the Root of the greater Number, divided by the Root of the leſſer. As the product of the cubical Numbers 343 upon 27 $=$ 9261 a Cube, whoſe Root is 7×3. Alſo $\frac{\surd qAqEq}{Bq}=\frac{AE}{B}$.

10. Whereſore the Multiplication and Diviſion of homogeneal ſurd Roots, produces a ſurd Root of the ſame kind: As $\surd q7$ upon $\surd q3$ is $\surd q21$ and $\surd q7)\ \surd q21\ (\surd q3$, or $\frac{\surd q21}{7}=\surd q3$. Alſo $\surd qA$ upon $\surd qE$, is $\surd qAE$ and $\surd qA)\ \surd qAE\ (\surd qE$, or $\frac{\surd qAE}{A}=\surd qE$.

11. But Roots of a different kind (or Heterogeneal) are not multiplied or divided, unleſs they be firſt reduced to the ſame kind, which is done by dividing the Indexes of both Powers propoſed, by their greateſt Com-

mon Meaſure. And multiplying the Index's by each others Quotients, and the Powers themſelves into the Species, denominated by each others Quotient. Suppoſe the $\surd qq10$ and $\surd cc7$ were to be multiplied or divided, firſt let them be reduced to the $\surd cccc1000$ and $\surd cccc49$, by cubing 10, and ſquaring 7; then multiply or divide. So $\surd qqA$ and $\surd ccBq$ are to be reduced to $\surd ccccAc$ and $\surd ccccBqq$. As you may ſee in the following Praxis.

$$\begin{array}{llllllll} & \surd & [12] & 1000 & \surd & [12] & 49 \\ [2]) & \surd & [4] & 10 & \surd & [6] & 7 \\ & & [2] & & & [3] & \end{array}$$

$$\begin{array}{llllllll} & \surd & [12] & Ac & \surd & [12] & Bqq \\ [2]) & \surd & [4] & A & \surd & [6] & Bq \\ & & [2] & & & [3] & \end{array}$$

Again, If the $\surd c32$ be to be multiplied by 2, for 2 take the $\surd c8$, and by it multiply $\surd c32$, and the Product will be $\surd c256 = 2 \times \surd c32$.

Alſo if the $\surd c32$ be to be divided by 2, for 2 take the $\surd c8$, and by it divide the $\surd c32$, and the quotient will be $\surd c4 = \frac{1\surd c32}{2}$.

So alſo $\frac{22}{7}\surd qAq = \surd q\frac{484}{49}Aq = \frac{22}{7}A$.

12. If the Root of a Power be to be muchplied, according to the Exigency of its own Species, blot out the Lateral Note of the the Species, as Q : $\surd q64$, or C : $\surd c64$ is 64.

13. And if the Root of a Power, whoſe Index is a compounded Number, be to be multiplied according to the exigence of either compounding Species. Let the Root of the other Species be prefix'd, the quantity remaining as it was. As
Q. $\surd cc64 = \surd c64$ and C. $\surd cc64 = \surd q64$, for $\surd cc = \surd\ [2 \times 3]$.

14. If a magnitude of ſeveral Parts or Names be drawn upon it ſelf, one of its Signs being changed, it will looſe one of them. As $3 + \surd 5 + \surd 2$ upon $3 + \surd 5 - \surd 2$ $= 12 + \surd 180$.

NOTES.

1. *'Tis plain, that* AMN. MNE : : Ac. Mc, *by contiuual multiplication of the ſeveral Terms as they ſtand over one another. And becauſe* AM N. MNE : : A.E; *(as is plain from the multiplication of* A *and* E *upon* MN *to make them)*; *therefore* A.E : : Ac. Mc.

7. Commenſurable Surds, *&c.*] *For as their Common Meaſure, multiplied upon theſe Numbers found, produce thoſe Surds, ſo the ſame Common Meaſure upon the ſum or difference of theſe Numbers found produces the Sum or Difference of thoſe Surds.*

10. Wherefore, *&c.*] *For ſuppoſe*

$$\begin{array}{l} A=bb \\ E=cc \end{array} \text{ and } \begin{array}{l} \sqrt{A}=\sqrt{bb}=b \\ \sqrt{E}=\sqrt{cc}=c. \end{array}$$

Then as $AE=bbcc$, ſo the Roots $\sqrt{A}\times\sqrt{E}$ $=bc=\sqrt{bbcc}=\sqrt{AE}$, and $\frac{\sqrt{A}}{\sqrt{E}}=\frac{b}{c}=\frac{\sqrt{bb}}{cc}$

11. *If any one does but conſider the Nature of Powers, he will find no great difficulty in reducing them.*

14. *You may ſet them thus, and multiply them.*

$$\begin{array}{l} \quad\;\; 3+\sqrt{5}+\sqrt{2} \\ \quad\;\; 3+\sqrt{5}-\sqrt{2} \\ \hline 9+3\sqrt{5} \\ \quad +3\sqrt{5}+\sqrt{25}-\sqrt{4} \\ \hline 9+6\sqrt{5}+:\sqrt{25}-\sqrt{4}:(=5-2=3.) \end{array}$$

That is $12+\sqrt{36}\times\sqrt{5}$,
Or $12+\sqrt{180}$.

CHAP. XVI

Concerning EQUATIONS.

1. WHenſoever any Problem is propoſed. Suppoſe the thing done, and having uſed a fit Ratiocination, call the ſought quantity A, or any other Vowel: and for the given Quantities put Conſonants.

2. Purſue the Queſtion as the matter will give leave, and compare, add, ſubſtract, multiply or divide, till at laſt you find ſomething given equal to the quantity ſought, or ſome power of it.

3. And becauſe at firſt the known and unknown Quantities lie promiſcuouſly together, the Terms are to be ſo ordered, that what is given in meaſure be brought to one ſide, and the unknown or ſought Quantities to the other. In doing which the five following Rules will direct.

4. Firſt, If the ſought Quantity; or any degree of it be in a Fraction, reduce all the Quantities to one Denomination, ſo that the common Denominator being dropt, the Equation may conſiſt only of the Numerators.

As $A - C = \frac{Aq + Bq}{D} + B + C$.

Then $DA - DC = Aq + Bq + DB + DC$.

5. Secondly, If what is given in meaſure be mixed with the unknown Quantities, tranſpoſe them both, changing their Signs.

$$DA - DC = Aq + Bq + DB + DC.$$

Tranſpoſe DC and Aq. $DA - Aq = 2DC + DB + Bq$. Which rule is to be obſerved in all Tranſpoſition.

6. Thirdly, If the higheſt Species of the ſought Quantity be drawn upon any given Quantity, divide the Equation by it.

$$BAq + BqA = Zc.$$

Divide by B. $Aq + BA = \frac{Zc}{B}$.

7. Fourthly, If it happen that all the given Quantities be drawn upon ſome degree of the ſought Quantity, divide the Equation by the loweſt Species, according to the order of the Table.

As $Aqq+BAc=ZqAq$.

Divide by Aq $Aq+BA=ZqAq$.

And after this manner any Equation may be depreſſed, if all the Terms be divided by the ſame degree; as $Ac+XAq=Nc$, divide by A, and there will be $Aq+XA=\frac{Nc}{A}$: But if you divide by Aq there will be $A+X=\frac{Nc}{Aq}$. Which Operation will be of uſe in the numerous Reſolution of affected Equations; becauſe the ſide is more eaſily judged of in leſſer than in greater Powers.

8. Fifthly, If any Quantity be a ſurd Root. The Equation is to be purſued in the Powers themſelves. $\surd qBA+B=C$. And by Tranſqoſition $\surd qBA=C-B$. Therefore being Squared,

$$BA=Cq-2CB+Bq, \text{ or}$$
$$A=\frac{Cq-2CB+Bq}{B}.$$

Alſo $\surd u:BA+CA:-D=B$, or $\surd u:BA+CA=D+B$. Therefore being ſquared, $BA+CA=Bq+2BD+Dq$, or $A=\frac{Bq+2BD+Dq}{B+C}$.

Laſtly, $\surd q\frac{A}{3}=\surd c2A$, or by the 11. of *Chap.* 15. $\surd cc\frac{Ac}{27}=\surd cc4Aq$. Wherefore $Ac=108Aq$, and $A=108$.

9. The conſtitution of Equations that conſiſt of three Species, equally aſcending in the Scale, will appear from *Sect.* 2, 3, 4. *Cap.* 11. For becauſe

$$\left.\begin{array}{r}Z-A=E\\ Z-E=A\\ A-X=E\\ E+X=A\end{array}\right\}\begin{array}{c}\text{Multiply}\\ \text{both}\\ \text{parts}\\ \text{upon}\end{array}\left\{\begin{array}{l}A\\ E\\ A\\ E\end{array}\right.$$

And ſo of the Z and X, *&c.*

And there will ariſe ſuch Equations.

$$\begin{array}{ll}ZA-Aq=\text{Æ} & Aq-XA=\text{Æ}\\ Z.Aq-Aqq=\text{Æ}q & Aqq-X.Aq=\text{Æ}q\\ ZAc-Acc=\text{Æ}c & Acc-XAc=\text{Æ}c,\\ \quad\textit{\&c.} & \quad\textit{\&c.}\end{array}$$

$$\begin{array}{ll}ZE-Eq=\text{Æ} & Eq+XE=\text{Æ}\\ Z.Eq-Eqq=\text{Æ}q & Eqq+X.Eq=\text{Æ}q\\ ZEc-Ecc=\text{Æ}c & Ecc+XEc=\text{Æ}c\\ \quad\textit{\&c.} & \quad\textit{\&c.}\end{array}$$

When therefore an Equation is propoſed, conſiſting of three Species, equally aſcending in the Scale. You muſt ſuppoſe the abſolute Quantity given, to be a Rectangle under the two ſought Quantities; Whether they be, Roots, or Squares, or Cubes, *&c.* or whatſoever be the Power of the middle Species. But in the middle Species, if the higheſt Species be negative, the Coefficient is the ſum of two unknown Quantities; and to be expounded of them both. If affirmative, the Coefficient is the difference of them: And the Species to be expounded of the greater, being negative, or of the leſſer, being affirmative.

Now the Sum and Rectangle of two Quantities be given, the Difference is alſo given: Or the Difference and Rectangle be given, the Sum is given. For by the 2, *Chap.* 11.

$$Q:\tfrac{1}{2}Z:-Æ=Q\tfrac{1}{2}X$$

$$Q:\tfrac{1}{2}X:+Æ=Q\tfrac{1}{2}Z$$

$$\text{Therefore } \surd u:\tfrac{1}{4}Zq-Æ=\tfrac{1}{2}X$$

$$\surd u:\tfrac{1}{4}Xq+Æ=\tfrac{1}{2}Z.$$

And $\frac{1}{2}Z$, and $\frac{1}{2}X$ of two Quantities being given, the Quantities themſelves are given.

I. Rule. $\frac{1}{2}2Z\pm\surd u:\frac{1}{4}Zq-Æ:(\frac{1}{2}X)=\frac{A}{E}$

II. Rule. $\surd u:\frac{1}{4}Xq+Æ:(\frac{1}{2}Z)\pm\frac{1}{2}X=\frac{A}{E}$

And theſe two Rules will ſerve to ſolve ſuch Equations as conſiſt of 3 Species equally aſcending in the Scale.

10. The Compoſition of the Six Binomials from their ſurd Roots. The Rule is this: $Z + 2\text{Æ} = Zq$. But for Reſiduals, 'tis $Z - 2\text{Æ} = Xq$.

I. Example. Let the Binomial $4 + \surd 11$ be Squared. Here the Z is $16 + 11 = 27$: and Æ is $\surd 16 \times \surd 11 = \surd 176$; the double of which is $\surd 704$. Therefore the Square will be $27 + \surd 704$. This is the firſt Binomial.

II. Example. Let the former Bimedial be Squared, *viz.* $\surd qq.\, 12 + \surd qq \tfrac{27}{4}$. Here Z is $\surd 12 + \surd \frac{27}{4}$, or $\surd \frac{48}{4} + \surd \frac{27}{4}$; that is, the $\surd \tfrac{147}{4}$ by 7. *Chap.* 15. And Æ is $\surd qq 12 \times \surd qq \tfrac{27}{4}$, or $\surd qq 3 \times \surd qq 27 = \surd qq 81$, *viz.* 3. the double of which is 6.

Therefore the Square will be $\surd \frac{147}{4} + 6$.

This is the ſecond Binomial.

III. Example. Let the latter Bimedial, be Squared, *viz.*

$\surd qq \frac{80}{3} + \surd qq 15$. Here Z is $\surd \frac{80}{3} + \surd .15$, or the $\surd \frac{80}{3} + \surd \frac{45}{3}$; that is $\surd \frac{245}{3}$ by 7 *Chap.* 15.

7, *Chap.* 15. And Æ is the $\sqrt{qq}\frac{80}{3}\times\sqrt{qq}$ 15, or the $\sqrt{qq}80\times\sqrt{qq}5$; that is, $\sqrt{qq}400$, *viz.* $\sqrt{20}$. The double of which is the $\sqrt{80}$.

Therefore the Square will be $\sqrt{\frac{245}{3}}+\sqrt{80}$. This is the Third Binomial.

In the Three following, which conſiſt of Binomial and Reſidual Roots connected. As $\sqrt{b}: A+E:$ pl $\sqrt{r}: A-E:$ It is manifeſt, that the Z is 2A, and the Æ is $\sqrt{Aq-Eq}$. Wherefore,

IV. Example. Let the Major be Squared, $\sqrt{b}: \frac{7}{2}+\sqrt{\frac{29}{4}}:$ pl. $\sqrt{r}.\frac{7}{2}-\sqrt{\frac{29}{4}}$. Here Z is $\frac{7}{2}+\frac{7}{2}$; that is 7. And Æ is $\sqrt{u}: \frac{49}{4}-\frac{29}{4}$ that is, $\sqrt{\frac{20}{4}}$, *viz.* $\sqrt{5}$. The double of which is the $\sqrt{20}$. Then the Square is $7+\sqrt{20}$. This is the Fourth Binomial.

V. Example. Let the Potent a Rational, with a Medial be Squared, *viz.* $\sqrt{b}: \sqrt{5}+1:$ pl. $\sqrt{r}: \sqrt{5}-1$. Here the ſum of the Squares is the $\sqrt{5}+\sqrt{5}$, that is $\sqrt{20}$; and Æ

$\AE$ is the $\surd 5 - 1$, that is, $\surd 4$, *viz.* 2. Therefore the Square is $\surd 20 + 4$. This is called the Fifth Binomial.

VI. Example. Let the Potent, two Medials be Squared, *viz.* $\surd b : \surd 5 + \surd 3$: pl. $\surd r. \surd 5 - \surd 3$. Here Z is the $\surd 5 + \surd 5$, that is $\surd 20$: and the Æ is the $\surd 5 - 3$, that is, $\surd 2$; the double of which is the $\surd 8$. The Square therefore is $\surd 20 + \surd 8$. This is called the Sixth Binomial.

II. *The Reſolution of* Binomials. In a Quadratick Binomial, the greater part or nomen is the Z, the leſſer 2Æ. But in the 2. *Chap.* 9. we have this Equation.

$$\frac{1}{4}Zq - \AE = \frac{1}{4}Xq : viz.\ \frac{1}{4}Q : A + E : - \AE = \frac{1}{4}Q : A - E.$$

Wherefore, if inſtead of A and E you take their Squares Aq and Eq, there will be $\frac{1}{4}Q : Aq + Eq : - AqEq = \frac{1}{4}Q : Aq - Eq$.

That is $\frac{1}{4}Zq - \AE q = \frac{1}{4}Xq$. From which

Theorem this Rule is deduced, for the Analyſis or Reſolution of Binomials.

$$\frac{1}{2}Z \pm \sqrt{q} : \frac{1}{4}Zq - \text{Æ}q : \left(\frac{1}{2}X\right) = \frac{Aq}{Eq}.$$

I. Example. Let the ſide of the firſt Binomial be ſought, *viz.* $27 + \sqrt{704}$; which conſiſts of theſe parts, $Z + 2\text{Æ}$. Wherefore $\frac{1}{2}Z$ is $\frac{27}{2}$, and Æ the $\frac{\sqrt{704}}{4}$, and $\frac{1}{4}Zq$ | Æq is $\frac{729}{4} - \frac{704}{4}$, that is $\frac{25}{4}$; the Root of which $\frac{5}{2}$ is $\frac{1}{2}X$. But by the Rule $\frac{27}{2} \pm \frac{5}{2} = \begin{matrix} 16. & 4 \\ 11. & \sqrt{11} \end{matrix}$: Therefore the Root is $4 + \sqrt{11}$. And iscalled the firſt Binomial.

II. Example. Let us find the Root of the ſecond Binomial, $\sqrt{\frac{147}{4}} + 6$, *viz.* $Z + 2\text{Æ}$. Wherefore $\frac{1}{2}Z$ is $\frac{147}{16}$, and Æ is 3. And $\frac{1}{4}Zq - \text{Æ}q$ is $\frac{147}{16} - (9)\frac{144}{16}$, that is, $\frac{3}{16}$.

Now the $\surd\frac{3}{16}$ is $\frac{1}{2}$X. But according to the Rule $\surd$:

$$\frac{147}{16}\begin{matrix}+\\-\end{matrix}\frac{\surd 3}{16}=\begin{matrix}\surd 12.\ \surd\, qq12\\ \surd\frac{27}{4}\cdot\ \surd\, qq\frac{27}{4}\end{matrix}$$

Therefore the Root is $\surd\, qq\,12+\surd\, qq\frac{27}{4}$. And it is called the former Bimedial.

III. Example: Let us find the Root of the third Binomial $\surd\frac{245}{3}+\surd 80$, *viz.* Z + 2Æ. $\frac{1}{2}$Z is the $\surd\frac{245}{12}$, and the Æ$\surd 20$ and $\frac{1}{2}$Zq—Æq is $\frac{245}{12}$— (20) $\frac{240}{12}$, that is, $\frac{5}{12}$. Now the $\surd\frac{5}{12}$ is $\frac{1}{2}$X. But by the Rule

$$\left.\surd\frac{245}{12}\begin{matrix}+\\-\end{matrix}\surd\frac{5}{12}=\begin{matrix}\surd\frac{80}{3}\ .\ \surd qq\frac{80}{3}\\ \surd 15\ .\ \surd qq15\end{matrix}\right\}$$

Therefore the Root of this Binomial is $\surd\, qq\frac{80}{3}+\surd qq15$. And it is called the latter Bimedial.

IV. Example. Let us find the Root of the fourth Binomial $7+\sqrt{20}$, *viz.* $Z+2Æ$. Wherefore $\frac{1}{2}Z$ is $\frac{7}{2}$; and Æ is the $\sqrt{5}$. and $\frac{1}{4}Zq - Æq$ is: $\frac{49}{4} - (5)\, \frac{20}{4}$: that is $\frac{29}{4}$: Now the $\sqrt{\frac{29}{4}}$ is $\frac{1}{2}X$.

But by the Rule

$$\frac{7}{2} \pm \sqrt{\frac{29}{4}} = \begin{array}{l} \frac{7}{2} + \sqrt{\frac{29}{4}}\sqrt{b} : \frac{7}{2} + \sqrt{\frac{29}{4}} \\ \frac{7}{2} - \sqrt{\frac{29}{4}}\sqrt{r} : \frac{7}{2} - \sqrt{\frac{29}{4}} \end{array}$$

Therefore the Root will be $\sqrt{b}.\ \frac{7}{2} + \sqrt{\frac{29}{4}}$: pl : $\sqrt{r}.\ \frac{7}{2} - \sqrt{\frac{29}{4}}$. And called the Major.

V. Example. Let us find the Root of the fifth Binomial $\sqrt{20}+Z$ *viz.* $Z+2Æ$. Wherefore $\frac{1}{2}\sqrt{20}$ is the $\sqrt{5}$: and Æ is 2; and $\frac{1}{4}Zq - Æq$ is $5-4$; that is, 1 : Whoſe Root 1 is $\frac{1}{2}X$. But by the Rule

$$\begin{array}{ll} \sqrt{5} \pm 1 = & \sqrt{5}+1 \,.\, \sqrt{b} \,.\, \sqrt{5}+1 \\ 5 & \sqrt{5}-1 \,.\, \sqrt{r} \,.\, \sqrt{5}-1. \end{array}$$

Therefore the Root is $\sqrt{b} : \sqrt{5}+1$: pl. $\sqrt{r}.\sqrt{5}-1$. And is called the Potent, a Rational with a Medial.

VI. Example. Let us find the Root of the ſixth Binomial $\sqrt{20}+\sqrt{8}$, *viz.* $Z+2Æ$.

Wherefore $\frac{1}{2}Z$ is $\surd 5$; and the Æ the $\surd 2$: and $\frac{1}{2}Zq - \text{Æ}q$ is $5-2$, that is 3; whoſe root $\surd 3$ is $\frac{1}{2}X$. But by the Rule

$$\surd 5 \pm \surd 3 = \begin{array}{l} \surd 4 + \surd 3 \,.\, \surd b.\, \surd 5 + \surd 3 \\ \surd 5 - \surd 3 \;\; \surd r.\, \surd 5 - \surd 3: \end{array}$$

Wherefore the Root of this Binomial is

$$\surd b.\, \surd 5 + \surd 3 : \text{pl.}\, \surd r\, \surd 5 - \surd 3.$$

And it is called the Potent two Medials.

12. And here by the way we may learn the Conſtruction of a plain Rectangular Triangle. Becauſe $Zq = Xq + 4AE$, *viz.* $Hq = Bq + Cq$. *per* 47 of the firſt Book of Euclid. Any two Lines or Numbers being given A and E, the ſides of the Rectangular Triangle will be $A+E$. $A-E$ and $\surd 4AE$, or (inſtead of A and E, you take Aq and Eq) they will be $Aq+Eq$. $Aq-Eq$. $2AE$. *Viz.* $\surd 4Aq\,Eq$ Or, if you take two Numbers, 2 and 1, the ſides will be 3.1. $\surd 8$. *viz.* $2+1$. $2-1$, $\surd 4\times2\times1$, or alſo 5, 3, 4 *viz.* $4+1$. $2-1$. and 2×1 twice.

13. Two Rectangular Triangles being given, H. B. C. and h. b. c. to make a third of them, and that two ways.

Firſt, Becauſe $Bq = Hq - Cq$,

and $bq = hq - cq$.

Let them be Multiplied upon one another and there will be $Bq\,bq = Hq\,hq + Cq\,cq$

mi. $Hqcq + Cq\,hq$.
But $Hqhq + Cq\,cq + 2HC\,hc = Q.Hh + Cc$.
And $Hqcq + Cqhq + 2HChc = Q.Hc + Ch$.

Subduct one Square from another and there will be $Bq\ bq = QHh + Cc$: mı. $Q: Hc + Ch$.

And ſo of theſe is made a third Triangle $Bb.\ Hh + Cc.\ Hc + Ch$. This is the firſt Rule. In Words thus:

For the Baſe of the new Triangle, take a Rectangle of the Baſes: For the Hypotenuſe a Rectangle of the Hypotenuſes increaſed, with a Rectangle of the Catheti. For the Cathetus: A Rectangle made by the Hypotenuſes of the firſt and the Cathetus of the ſecond increaſed with a Rectangle, made by the Cathetus of the firſt, and the Hypotenuſe of the ſecond.

Secondly, Becauſe
$$\begin{aligned} Hq &= Bq + Cq \\ hq &= bq + cq. \end{aligned}$$
Multiply them upon one another; there will be $Hq\ hq = Bq\,bq + Cq\ cq$ pl. $Bq\,cq + Cq\ bq$.

But $Bq\ bq + Cq\ cq - 2BC\,bc = Q.Bb - Cc$.
And $Bqcq + Cq\ bq + 2BC\,bc = Q.Bc + Cb$.
Add theſe together, and there will be $Hq\ hq = Q{:}Bb - Cc{:}$pl. $Q.\ Bc + Cb$ And ſo of theſe is made a 3d. Triangle, $Hh.Bb - Cc.Bc + Cb$.

This is the ſecond Rule. In Words thus.

For the Hypotenuſe of the new Triangle take the Rectangle of the Hypotenuſes

For the Baſe; a Rectangle of the Baſes leſs by the Rectangle of the Catheti. For the Cathetus; a Rectangle made by the Baſe of the firſt, and the Cathetus of the ſecond, increaſed with a Rectangle made by the Cathetusof the firſt and the Baſe of the ſecond.

14. If the ſides of a Rectangular Triangle be Multiplied continually according to theſe two Rules, the firſt Multiplication will make a Bicompoſit Triangle, the ſecond a Tricompoſit, the third a Quadricompoſit, *&c.*

Example. Rule I. Bb. Hh + Cc. Hc + Ch.

B. H. C. A ſimple Triangle.

B. H. C.

Bq. Hq + Cq. 2HC. A Bicomoſit Triangle.

B H C

Bc. Hc + HCq. 2HqC

2HCq HqC + Cc

Bc.Hc + 3HCq 3HqC + Cc. Tricompoſit

B H C

Bqq. Hqq + 3HqCq. 3HcC + HCc

Cqq + 3HqCq. HcC + 3HCc

Bqq. Hqq + 6HqCq + Cqq. 4HcC + 4HCc.

B H C, *&c.*

Quadricompoſit.

Example. Rule II. Hh. Bb—Cc. Bc+Cb.

$$
\begin{array}{llll}
\text{H. . B. C.} & \text{Simple.} & & \\
\text{H. B. C} & & & \\
\hline
\text{Hq. Bq—Cq. 2BC.} & \text{Bicompoſit.} & & \\
\text{H} & \text{B} & \text{C} & \\
\hline
\text{Hc. Bc—BCq. 2BqC} & & & \\
\quad\text{—2BCq BqC - Cc} & & & \\
\hline
\text{Hc. Bc—3BCq. 3BqC—Cc.} & \text{Tricompſit.} & & \\
\text{H} & \text{B} & \text{C} & \\
\hline
\text{Hqq. Bqq—3BqCq. 3BcC—BCc} & & & \\
\quad\text{Cqq—3BqCq. BcC—3BCc} & & & \\
\hline
\text{Hqq. Bqq—6BqCq+Cqq. 4BcC—4BCc.} & & & \\
\text{H} & \text{B} & \text{C, \&c.} & \text{Quadricompoſit.} \\
\hline
\end{array}
$$

NOTES.

9. In which there are Three Species, *&c.*] *That is, if the Exponents of the unknown Quantity be in Arithmetical Progreſſion. As* AA.A. 1. *Whoſe Exponents are* 2. 1. 0. *or* Aqq. Aq. 1. *whoſe Exponents are* 4. 20. *and the like.*

Now the Sum and Rectangle, *&c.*] *For becauſe* Q. $\frac{1}{2}$Z : —Æ=Q. $\frac{1}{2}$X. *Or which is all one,* $\frac{1}{4}$Zq : —Æ=$\frac{1}{4}$Xq. Wherefore √u : $\frac{1}{4}$Zq—Æ=√$\frac{1}{4}$Xq=$\frac{1}{2}$X.

Again, $Q\,\frac{1}{2}X : + Æ = Q.\,\frac{1}{2}Z$. *Or which is all one,* $\frac{1}{4}Xq + Æ = \frac{1}{4}Zq$. Therefore $\surd u : \frac{1}{4}X + Æ = \surd\,\frac{1}{4}Zq = \frac{1}{2}Z$.

10. *'Twill not here be a miſs to take an Example or two.*

The Fifth Example is this.

$\surd b : \surd 5 + 1 :$ pl. $\surd r.\, \surd 5 - 1$, and (as hath been obſerved) $Z = 2A$ and $Æ = \surd Aq - Eq$. Therefore the $Z = 2\surd 5$; that is the $\surd 20$. (For ſuppoſe 20 in Square $\begin{array}{|c|c|}\hline 5 & 5 \\ \hline 5 & 5 \\ \hline\end{array}$ the $\surd 5$ will be half the $\surd 20$, and $2\surd 5$ the $\surd 20$; as appears from the Figure.) And the $Æ = \surd 5 - 1$. For (as was before ſaid) $Æ = \surd Aq - Eq$. and $5 = Aq$ and $1 = Eq$. Therefore $\surd 5 - 1 = \surd Aq - Eq$.

The Sixth Example is this.

$\surd b : \surd 5 + \surd 3 :$ pl. $\surd 5 - \surd 3$. Here again for the ſame reaſon, Z is $2\surd 5$; that is, $\surd 20$: and the $Æ$ is $\surd 5 - 3$; for $5 = A\,A$ and $3 = EE$.

CHAP. XVII.

A Second Inſpection of the latter Table *in the* 12. *Chap.*

1. ALL the Species of Powers ariſing from the Binomial Root $A+E$ are Affirmative. Every other Species of Powers, ariſing from the Reſidual Root, is Negative.

As the Q : $A-E$: is $Aq-2AE+Eq$, and the C: $A-E$: is $Ac-3AqE+3AEq-Ec$. And the QQ : $A-E$: $Aqq-4AcE+6Aq\,Eq-4AE\,c+Eqq$, *&c.* So that if you take the Alternate Species of any Power and make two diſtinct Sums of them, theſe Sums connected with the Sign of the Root, will be the Power of the Root it ſelf. And this is the Conſtitution of Binomials and Reſiduals, Quadratick and Cubick, *&c.*

2. Wherefore the Difference of the Parts of a Binomial or Reſidual Power is the homogene Power of the Difference of the parts of the Root. As $Ac+3AEq$ mi. $3AqE+Ec$, or $Ac+3AEq-3AqE-Eq=C.A-E$.

3. And the Difference of the Squares of the parts of a Binomial or Reſidual Power, is the homogene power of the Difference of the Squares of the Parts of the Root. As

Q: Ac+3AEq mi. Q. 3AqE+Ec is C: Aq—Eq.

For according to the Example of Rule I. in the 16. *Chap. Sect.* 14. If A be the Hypotenuſe of the Triangle, and E the Cathetus, and Aq—Eq the Square of the Baſe. The Theorem may otherwiſe be expreſſed thus,

Q. Hc+3HCq: mi: Q. 3HqC+Cc= C: Bq=Q: Bc. Therefore, *&c.*

4. But if the Species of which thoſe Parts conſiſt, be alternately Affirmative and Negative, the Sum of the Squares of thoſe parts is the homogene Power of the Sum of the Squares of the parts of the Root.

As Q. Ac—3AEq. pl. Q. 3AqE—Ec is the C: Aq+Eq.

For according to the Example of Rule II. *Chap.* 16. *Sect.* 14. If A be the Baſe of the Triangle, and E the Perpendicular, and Aq+Eq the Square of the Hypotenuſe, the Theorem may otherwiſe be expreſſed thus;

Q. Bc—3BCq. pl. Q. 3BqC—Cc= =C. Hq=Q. Hc. Therefore, *&c.*

5. All the Intermediate Species of every Rank, are alſo Powers of mean Proportionals between A and E. For between Ac and Ec there are two mean Proportionals, AqE and AEq; which are alſo Cubes of M and N: So that A. √cAqE. √cAEq. E. are in continual Proportion, *viz.* A.M.N.E.

For AqE=AMN=Mc and AEq=MNE =Nc. And from hence you may learn to find as many mean Proportions as you pleaſe. If you would find Five, the Power muſt be the Sixth, or cc; the Index of which exceeds, by one, the number of mean Proportionals ſought.

And they will be A. √ccAqcE. √ccAqq. Eq. √ccAcEc. √ccAqEqq. √ccAEqc. E. ∺.

6. Every intermediate Species is made of the two Powers of the Parts of the Root, whoſe Indexes both together are equal to the Index of that Power to which it belongs. But the diſtance of the middle Species from the Extremes, is equal to the Index of one of the foregoing Species on the one hand, and the other on the other of which it is Compounded; and between which it ſtands, *viz.* AqEc is made by Aq upon Ec, and is the third from Aqc, and the ſecond from Eqc.

A Conſectary. And hence it is eaſie to conſtruct the Power of any Binomial Root given, having found all the mean Proportionals between each part of the Root, and that Power of each which you are to Conſtruct. For Example; Let the fifth Power of A+√Æ be Conſtructed, the Cube will be

Aqc +
10ÆAc +
5ÆqA *plus*
5√ÆAqcc +
10√ÆcAqq +
√Æqc.

Aqc
Ac Aqq 5√ÆAqcc
A Aq 10ÆAc
10√ÆcAqq
√Æ Æ 5ÆqA
√Æc Æq √ÆqAc.

Which is a Binomial Quadrato-Cubical.

7. If any Species be multiplied upon the Æ, the Product will be a middle collateral Species, in the next row ſave one, and equally diſtant in number from the two Extremes.

Ac×Æ is AqqE, which is the firſt in order after Aqc, and the fourth from Eqc. So AcE×Æ is AqqEq, which is the ſecond from Acc, and the fourth from Ecc: And ſo of the reſt.

8. If any Species be multiplied by A−E or X, the product will be the Difference between thoſe two Species in the row following it that lie next it, one on one ſide, and the other on the other. As

AcX=Aqq−AcE. AqEX=AcE−AqEq.
AEqX=AqEq−AEc. EcX=AEc−Eqq.

Hence if all the Species of any Rank be multiplied by X, the Difference of the two extreme Species of the next higher Rank will be produced.

As $Ac+AqE+AEq+E\times X$ is $Aqq-Eqq$.

9. In thoſe Ranks which ſtand in odd places, or have odd Indexes, (*l,c, q c,* &c.) the ſum of the two extreme Powers: But in thoſe Ranks which have even Indexes (*q, q'q cc.*&c.) the Difference of the ſame is made by $A+E$, drawn upon each Species of the lower foregoing Rank, having their Signs alternately, affirmative, and negative.

As $Ac+Ec$ is made by $Aq-AE+Eq$ drawn upon $A+E$. And $Aqq-Eqq$ is made by $Ac-AqE+AEq-Ec$ upon $A+E$.

10. If the ſame Quantity be multiplied upon two contrary Quantities, the produced Quantities will be contrary. As $Aq-2AE+Eq$ upon $A-E$ makes $Ac-3AqE+3AEq-Ec$. But the ſame upon $-A+E$ makes $-Ac+3AqE-3AEq+Ec$.

11. The Unciæ or Numbers perfixed to the ſeveral Species are numerarie Figures. All under A and E are laterals; all under Aq and Eq are Triangulars; all under Ac and Ec are Pyramidals; all under Aqq and Eqq are Triangulo-Triangulars; all under Aqc and Eqc are Triangulo Pyramidals; all un-

der Acc and Ecc are Pyramidi-Pyramidals, *&c.*

12. If a Root conſiſts of three parts A.E.I.

$$\text{The Square}\left\{\begin{array}{l}Aq, 2AE\\ Eq, 2EI\\ Iq, 2AI\end{array}\right\}\text{Cube}\left\{\begin{array}{l}AcEc.\ Ic\\ 3AqE.\ 3AEq\\ 3AqI.\ 3AIq\\ 3EqI.\ 3EIq\\ \quad 6AEI.\end{array}\right.$$

And Note, that if any Species, the number of negative Sides be odd, the Species will be negative; as $Q: A+E-I$: is $Aq+2AE+2AE+Eq-2EI+Iq-2AI$, and the $C: A+E-I$ is $Ac+3AqE+3AEq+Ec-3EqI+3EIq-Ic-3AqI+3AIq-6AEI$.

NOTES.

3. *If you Square* $Ac+3AEq$, *and Square* $3AqE+Ec$, *and ſubſtract the latter from the former, you will then find the remainder equal to the Cube of* $AA-EE$.

5. Which are alſo Cubes of M and N] *Let there be* A.M. N.E ∺, *then there will be* $AMN=Mc$, *and* $MNE=Nc$. *For* $AN=Mq$, *and therefore* $ANM=Mc$ *and*

$ME=Nq$, *and therefore* $MNE=Nc$. *So here multiply* A *and* $\surd cAqE$ *and* $\surd cAEq$ *continually; or which is all one,* $\surd cAAA$ and $\surd cAqE$ and $\surd cAEq$ *continually, and there will ariſe* $\surd cAAAAAAEEE=AAE$. *Likewiſe multiply* $\surd cAqE$ *and* $\surd cAEq$ *and* $\surd cEEE$ *continually, and there will ariſe* $\surd cAAAEEEEEE=AEE$. *Therefore they are in continual Proportion.*

CHAP. XVIII.

Of EQVATIONS.

1. FRom the firſt and moſt eaſie Equations, which are nothing but either expoſitions of the Terms or ſimple Affections, (ſuch as are thoſe of Chap. 11. $\frac{1}{2}Z-E=\frac{1}{2}X$, and $\frac{1}{2}x+E=\frac{1}{2}Z$, *&c.*) innumerable others may be deduced by Addition, Subduction, Multiplication, Diviſion, Tranſpoſition, and Interpretation; by taking that which is found equal to ſomething elſe, in the room of that to which it is found equal. Which Analytical Furniture is not leſs precious than plentiful; of which I ſhall give you ſome of the moſt chief and neceſſary ones. Many more may be found by a Student of this Art. And whereſoever, either

in Arithmetick, or Geometry, or in any other Art, he ſhall meet with a quantity equal to another, that is the ſame quantity diverſly expreſſed. He muſt all manner of ways diſcuſs and vary that equality, that he may find a new Inſtrument that may be uſeful to him, which he is to keep in ſtore, and (as he ſhall have opportunity) to bring forth for the help and advancement of this Art.

2. $$\begin{array}{ll} Q:1:=9Q:\tfrac{1}{3}:\&c. & C:1=27\,C:\tfrac{1}{3}: \\ Q:1:=\tfrac{1}{9}Q:3:\&c. & C.1:=\tfrac{1}{27}\,C:3: \\ Q:1.=\tfrac{9}{4}Q:\tfrac{2}{3}:\&c. & C.1.=\tfrac{27}{8}\,C:\tfrac{2}{3}: \\ \tfrac{2}{3}Q:1=\tfrac{2}{3}\times\tfrac{9}{4}Q:\tfrac{2}{3}:\&c. & \tfrac{2}{3}C:1:=\tfrac{2}{3}\times\tfrac{27}{8}C:\tfrac{2}{3}: \\ \tfrac{2}{3}Q.4:=\dfrac{2\times16Q:1}{3}:\&c. & \tfrac{2}{3}C.4:=\dfrac{2\times64C:1}{3} \end{array}$$

3. If a Line be biſected and otherwiſe; the Rectangle under the unequal Segments, is equal to the difference of the Squares of the Biſegment and the Interſegment; *i. e.* of half the ſum and half the difference of the Segments, 5 è 2. $AE=Q:\tfrac{1}{2}A+\tfrac{1}{2}E:$ mi $Q:\tfrac{1}{2}A-\tfrac{1}{2}E$, and that is $AE=\tfrac{1}{4}Zq-\tfrac{1}{4}Xq$.

4. If a Biſected Line be increaſed. A Rectangle under the whole increaſed, and the augment is equal to the difference of the

Squares of the Biſegment increaſed, and the Biſegment 6 è 2. $A+E$ upon $E=Q: \frac{1}{2}A+E: \text{mi } Q: \frac{1}{2}A$. And $A+E$ upon $A=Q: \frac{1}{2}E+A: \text{mi } Q: \frac{1}{2}E$.

Therefore the ſum of three ∺, $(Aq+AE+Eq)$ with either of the Extrems being given, the other two are given thus:

$$\sqrt{u}: Aq+AE+Eq-\tfrac{3}{4}Aq : \text{mi } \tfrac{1}{2}A=E$$

$$\sqrt{u}: Aq+AE+Eq-\tfrac{3}{4}Eq : \text{mi } \tfrac{1}{2}E=A.$$

For $Q: \frac{1}{2}A+E: =\frac{1}{4}Aq+AE+Eq$.
And $Q: \frac{1}{2}E+A=Aq+AE+\frac{1}{4}Eq$.

5. If you divide a Line any where, the Sum of the Squares of the whole, and of one Segment, is equal to the aggregate of the Square of the other Segment, and a double Rectangle under the whole Line, and the former Segment, 7 *è* 2. $Zq+Aq=2ZA+Eq$, and $Zq+Eq=2ZE+Aq$. Wherefore $2ZA+Eq-Aq=Zq=2ZE+Aq-Eq$.

6. If a Line be divided any where and lengthened with either of the Segments, four Rectangles under the Line ſo divided, and the added Segment is equal to the Difference between the Square of the whole line, ſo lengthened, and the Square of the other Segment, 8 *è* 2. $Q: Z+E: -Aq=4ZE$, and $Q: Z+A: -Eq=4ZA$.

7. If a Line be divided into equal and into unequal Segments, the Sum of the Squares of the unequal Segments, is equal to twice the Sum of the Squares of the Biſegment and Interſegment, 9 è 2.
$Aq + Eq = 2Q : \frac{1}{2}A + \frac{1}{2}E : + 2Q . \frac{1}{2}A - \frac{1}{2}E .$

8. If you divide a Line equally and add a Line to it. The Sum of the Squares of the whole, ſo lengthened, is equal to twice the Sum of the Squares of the Biſegment ſo lengthened, and the Biſegment 10 è 2.
$Q : A + E : + Eq = 2Q : \frac{1}{2}A + E : + 2Q : \frac{1}{2}A :$
$Q : A + E : + Aq = 2Q : \frac{1}{2}E + A : + 2Q : \frac{1}{2}E :$

9. $Aq = ZA - AE = XA + AE = \frac{1}{2}ZA + \frac{1}{2}XA = Q : Z - E : = Q : E + X : = \mathcal{Z} - Eq = Eq + X .$
And $Eq = ZE - AE = AE - XE = \frac{1}{2}ZE - \frac{1}{2}XE = Q : Z - A : = Q . A - X = \mathcal{Z} - Aq = Aq - X .$

10. $\text{Æ} = \frac{1}{4}Zq - \frac{1}{4}Xq = ZA - Aq = ZE - Eq = Aq - XA = Eq + XE = \frac{1}{2}Zq - \frac{1}{2}\mathcal{Z} = \frac{1}{2}\mathcal{Z} - \frac{1}{2}Xq - \frac{1}{2}ZA - \frac{1}{2}XA = \frac{1}{2}ZE + \frac{1}{2}XE .$

11. $\mathcal{Z} = Aq + Eq = Zq - 2AE = 2AE + Xq = ZE + XA = ZA - XE = 2Q : \frac{1}{2}Z : + 2Q : \frac{1}{2}Z - E : = QA - 2N : + Q : 2M - E = \frac{1}{2}Zq + \frac{1}{2}Xq = 2Q : \frac{1}{2}Z : + 2Q : \frac{1}{2}X : .$ This

Conſectary may be drawn from the two laſt Equations, *viz.* If a Quantity conſiſt of the Squares of two other Quantities, its double will conſiſt of two Squares, namely, of the Sum and of the Difference. And its half will conſiſt of two Squares, namely, of half the Sum, and half the Semi-Difference.

And $\mathcal{X} = Aq - Eq = ZX = 2ZA - Zq = Zq - 2ZE = 2XA - Xq = 2XE + Xq = ZA - ZE = XA + XE = Zq - 2ZE = ZA + XE - 2\text{Æ} = XA + 2\text{Æ} - ZE = Q : A + 2N : mi : Q.\ 2M + E.$

12. $Q : \frac{1}{2}A + \frac{1}{2}E : = Q : \frac{1}{2}A - \frac{1}{2}E : + \text{Æ}.$ And $Q.\ \frac{1}{2}A - \frac{1}{2}E = Q.\ \frac{1}{2}A + \frac{1}{2}E : - \text{Æ}.$ For $\frac{1}{4}Zq = \frac{1}{4}Xq + \text{Æ}.$

13. $2A + 2E$ upon $A = 2Aq + 2AE = Zq + \mathcal{X}q.$ And $2A - 2E$ upon $A = 2Aq - 2AE = \mathcal{X} + Xq.$ And $2A + 2E$ upon $E = 2AE + 2Eq = Zq - \mathcal{X}.$ And $2A - 2E$ upon $E = 2AE - 2Eq = \mathcal{X} - Xq.$

14. $\mathcal{X}q = ZqXq = \mathcal{Z} + 2\text{Æ}$ upon $\mathcal{Z} - 2\text{Æ} = \mathcal{Z}q - 4AqEq.$

15. $Z\text{Æ} = AqE + AEq.$ And $X\text{Æ} = AqE - AEq.$ And $\mathcal{Z}\text{Æ} = AcE + AEc.$ And $\mathcal{X}\text{Æ} = AcE - AEc.$ Wherefore $\mathcal{Z} + 3Z\text{Æ} = Zc.$ And $\mathcal{Z} - 3X\text{Æ} = Xc.$ So that $\mathcal{Z}Z = \mathcal{Z}$

=𝒵+ZÆ=Ac+AqE+AEq+Ec. And zX=𝒳−XÆ=Ac−AqE+AEq−Ec. And xZ=𝒳+XÆ=Ac+AqE−AIq−Ec. And xX=𝒵−ZÆ=Ac−AqE−AEq+Ec. Hence zZ+xX=2𝒵. And xZ+zX=2𝒳. And zZ−xX=2ZÆ. And xZ−zX=2XÆ.

16. If in a Circle 7 be to 22 : : δ.π : : 113. 355. then
δ.π : : 2R. P. Periph.
δ.π : : Rq. Circle.
δ.π. : : 2Rc. Cylind.
δ.π : : $\frac{4}{3}$Rc. Sphær.
δ..π : : $\frac{2}{3}$Rc. Cone.

And

π.δ : : $\frac{1}{2}$P. R. Semidiam.
π.δ : : $\frac{1}{4}$Pq. Circle.
πq.δq : : $\frac{1}{4}$Pc. Cylind.
πq.δq : : $\frac{1}{6}$Pc. Sphær.
πq.δq : : $\frac{1}{12}$Pc. Con.

17: Moreover theſe Problems and Theorems are to be underſtood.

Theorm 1. Thoſe Triangles are equal, in which either three Sides, or two with the Angle made by them, or two Sides with an Angle oppoſite to the ſame Side, ſuppoſing there be an Angle oppoſite to the third Side of the ſame kind: Or two Angles with the Side that lies between them, or two Angles with a ſide ſubtended to the ſame Angle, are equal. 4. 8. 26. *è* 1.

Theor. 2. Plain Triangles are like, if they are Equiangular, or have all their ſides proportional, or have one Angle equal, and another between proportional Sides, and a third of the ſame kind; 4. 5. 6. 7. *è* 6.

Theor. 3. In every Triangle the greater Side is ſubtended to the greater Angle, and the leſſer to the leſſer, and the equal ſide to the equal. 18. 19. *è* 1.

Theor. 4. Two right Lines are Parallel, if a right Line cutting them makes either the alternate Angles, or the external and internal, oppoſite Angles, or the two internal of the ſame ſide, equal to two right Angles. And on the contrary, 27. 28. 29. 30. *è* 1. For parallel Lines are as it were one broad Line.

Theor. 5. The three Angles of a Triangle are equal to two right Angles; and the external Angle equal to the two internal, and oppoſite Angles, 32. *è* 1.

Theor. 6. If a right Line drawn from the Center, cut a right Line inſcribed in the Circle in half, it will alſo cut it at right Angles, 3. *è* 3.

Theor. 7. A Perpendicular to the end of the Diameter, is a Tangent to the Circle by the 16. 18. 19. *è* 3.

Theor. 8. The Angle at the Center is equal to twice the Angle at the Circumference.

Theor. 9. In the ſame Circle or in equal Circles, the Angles upon equal Arches are equal, 21 *è* 3.

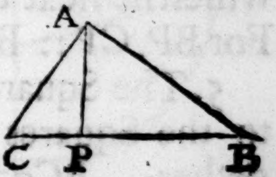
Theor. 10. An Angle in a Semicircle is a right Angle; 31 *è* 3.

Theor. 11. If from a point of the Circumference two right Lines be drawn, one touching the Circle, the other cutting it. The meaſure of the Angle comprehended between them, will be equal to half ſo much of the Circumference as is cut of, 32 *è* 3.

Theor. 12. Triangles or Paralellograms of the ſame heighth, or between the ſame parallel Lines are to one another as their Baſes, 35, 36, 37, 38, *è* 1. And 1 *è* 6.

Theor. 13. If a right Line divide equally an Angle of a Triangle, it will cut the Baſe into parts proportional to the Legs, 3 *è* 6.

Theor. 14. Let any Rectangular Triangle be noted, with the Letters A B C. So that A ſtands at the right Angle, and BA may denote the Baſis, and CA the Cathetus, and BC the Hypotenuſe.



Theor. 15. In ſuch a Triangle, a Perpendicular from the right Angle upon the Hypotenuſe, divides the Triangle into two Triangles like to the whole, and to one another. 8 è 6.

$$\underbrace{BC.\ BA.\ CA}_{\text{The Hypot.}} :: \underbrace{BA.\ BP.\ AP}_{\text{The Baſes.}} :: \underbrace{CA.\ AP.\ CP.}_{\text{The Perpend.}}$$

Hence, 1. the Perpendicular is a mean proportional between the Segments of the Hypotenuſe, and therefore the Square of the Cathetus is equal to a Rectangle made by the Segments. *Viz.*

∺ BP. AP. CP. and $APq = BP \times CP$.

2. The Baſe is a mean Proportional between the Hypotenuſe, and that Segment of it, which lieth next the Baſe, *viz.*

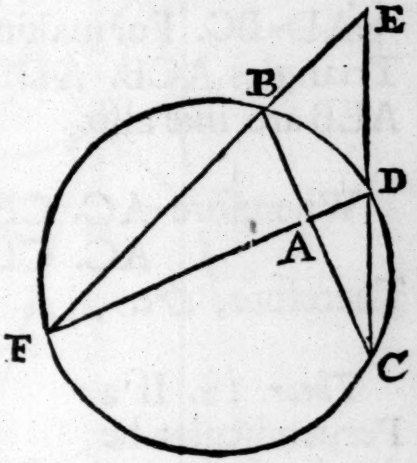
∺ BC. BA. BP.

3. The Perpendicular is a mean proportional between the Hypotenuſe, and that Segment of it which lieth next the Cathetus, *viz.* ∺ BC. CA. CP.

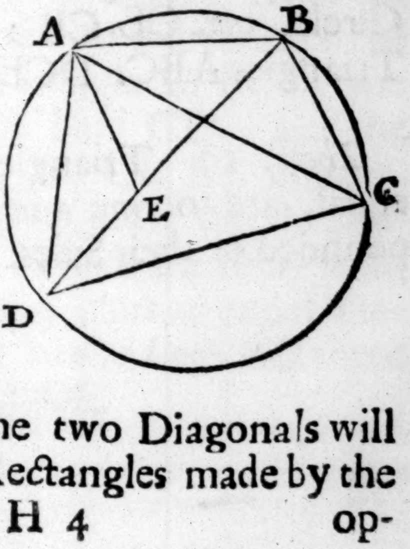
4. The Square of the Baſe and Cathetus, are as the Segments of the Hypotenuſe which lie next them, BP. CP :: BAq. CAq. For BP. CP :: $BC \times BP$. $BC \times CP$ :: BAq. CAq.

5. The Squares of the Hypotenuſe is equal to the Squares of the Baſe and Cathetus together. $BCq = BAq + CAq$. For $BCq = BC \times BP + BC \times CP = BAq + CAq$.

Theor. 16.



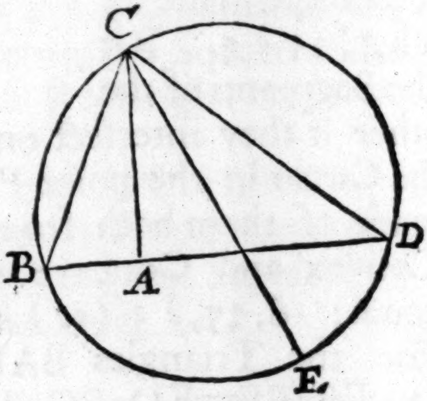
Theor. 16. If two right Lines inſcribed in the Circle, interſect one another within the Circle, in the Point (A.) A Rectangle made by the Segments of the one, is equal to a Rectangle made by other, 35 *è* 3. But the Segments of the other if they interſect one another without the Circle in the point E. The Rectangles made of them both from the point to the Convex and Concave of the Circle are equal; 36, 37, *è* 3. (1.) AB×AC=AD×AF. For the Triangles BAF. DAC. are like. (2) EB×EF=ED×EC. For the Triangles BEC. DEF are like.



Theor. 17. The inward oppoſite angles of a four ſided Figure inſcribed in a Circle, are both together equal to two right Angles. And if you draw two Diagonals, the Rectangle made by the two Diagonals will be equal to the two Rectangles made by the oppoſite Sides. I ſay, $AC \times BD = AB \times CD + AD \times BC$. For making $DAE = CAB$. The Triangles ACB. ADE are like, and ADC. AEB are like alſo.

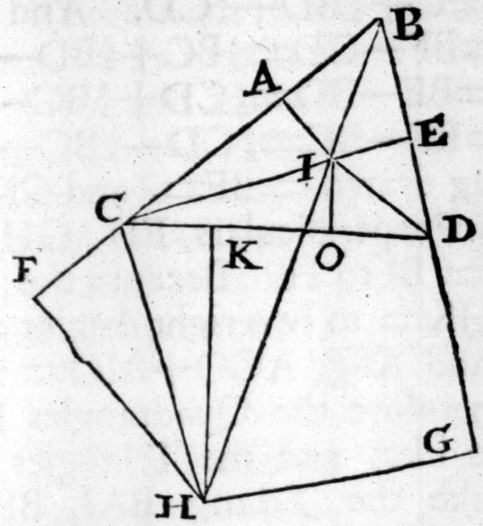
Wherefore AC. CB : : AD. DE
AC. CD : : AB. BE.

Therefore, *&c.*



Theor. 18. If a Perpendicular be drawn from any Angle of a Triangle inſcribed in a Circle to the oppoſite ſide, as that Perpendicular is to one Leg of the ſame angle, ſo the other Leg is to the Diameter of the Circle, *viz.* CE. CB : : CD. CE. For the Triangles ABC. DCE. are like.

Theor. 19. Triangles having one Angle equal, are to one another in a Ratio compounded of their Sides. 23. è 6.



Theor. 20. If (any ſide of a Triangle being taken for the Baſe, and the other two ſides for the Legs) a Rectangle made by half the Sum and half the Difference of the Sum of the Legs, and the Baſe be drawn upon a Rectangle made by half the Sum and half the Difference of the Baſe, and the Difference of the Legs. The Square Root of the Product will be equal to the Area of the Triangle. Let the Triangle be BCD, the Legs BC and BD, the Baſe CD. Let the 3 Angles be Biſected, BI. CI. DI. concurring in I. From whence draw the Perpendiculars IA. IE. IO. There are then within the Triangle BCD, 3 pairs of equal Triangles. .Wherefore if to the Legs BC there be added directly CF=DE.

There will be

$$BF = \tfrac{1}{2}C + \tfrac{1}{2}BD + \tfrac{1}{2}CD.\ \text{And}$$
$$BA = BF - CD = \tfrac{1}{2}BC + \tfrac{1}{2}BD - \tfrac{1}{2}CD$$
$$AC = BF - BD = \tfrac{1}{2}CD + \tfrac{1}{2}BC - \tfrac{1}{2}BD$$
$$CF = BF - BC = \tfrac{1}{2}CD - \tfrac{1}{2}BC + \tfrac{1}{2}BD$$

BG being drawn $=BF=$. and $CK=CF$. Draw the Perpendiculars FH, GH, KH: and produce BI to H. Becauſe the Angles $FCK+FHK=$ to two right Angles $=FCK+ACO$, and Ang. $ACO+AIO=2$ right Ang. Therefore the Quadrangles FCKH AIOC are like, and the Triangles CFH, IAC are like the Triang. BAI, BFH are like alſo. Now I ſay, that the Square of the Area of the Triangle, *viz.* $BFq \times IAq = BF \times BX \times AC \times CF$. For

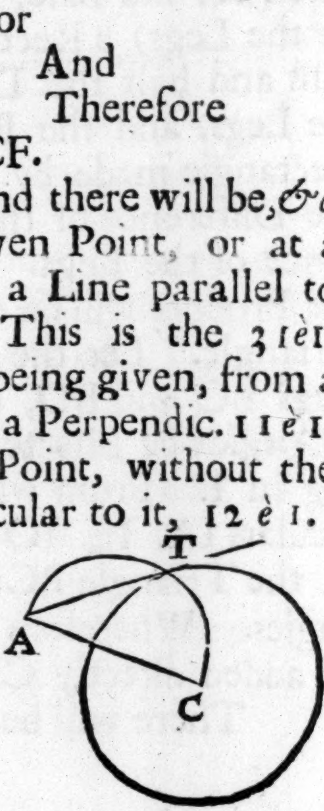
$$IA.\ BA :: FH.\ BF.\ \text{And}$$
$$IA.\ AC :: CF.\ FH.\ \text{Therefore}$$
$$IAq \times BF = BA \times AC \times CF.$$

Now Multiply by BF, and there will be, &c.

Probl. 1. From a given Point, or at a diſtance given, to draw a Line parallel to a right Line given. This is the 31 *è* 1.

Prob. 2. A right Line being given, from a given point in it, to erect a Perpendic. 11 *è* 1.

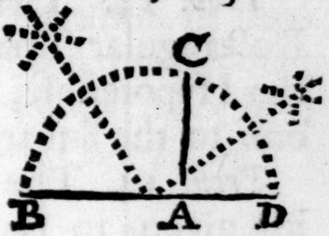
Prob. 3. From a given Point, without the Line, to drop a Perpendicular to it, 12 *è* 1.



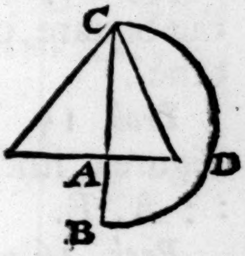
Prob. 4. From the point given A, without the Circle C, to draw a Tangent AT. 17 *è* 3.

Prob. 5. Three right Lines being given, to find a fourth Proportional, 12 *è* 6.

Prob. 6. Two right Lines AB, AD, being given, to find a mean Proportional, 13 *è* 6.

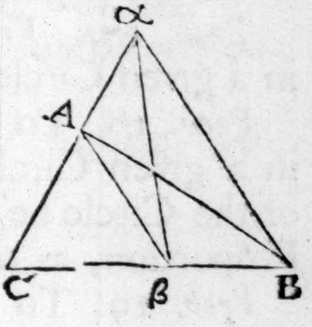


Prob. 7. Two right Lines AB, AC, or AD, AC being given, to find a third in continual Proportion, 11 *è* 6.

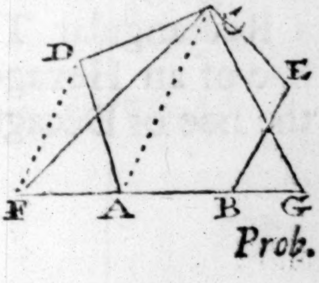


Prob. 8. A Triangle being given, whoſe heigth is AC, and whoſe half Baſe is AB, to make ADqeqnal to it.

Prob. 9. A Rectangle being given, to make another Rectangle equal to it, with a given ſide, 14 *è* 6.



Prob. 10. A Triangle being given, to make another equal to it, of a given heigth. From the heighths A and α, let the parallel Lines Aβ and αB be drawn to the oppoſite Angles.



Prob. 11. A Polygon being given, to make a Triangle equal to it.

Prob. 12. Three points being given, not lying in a ſtreight Line, to draw a circumference through them. 25 *è* 3.

Prob. 13. The Baſe and Cathetus of a Rectangular Triangle being given, to find the Hypotenuſe, or to add the Square of one to the Square of the other.

Prob. 14. The Hypotenuſe and Baſe being given, to find the Cathetus, or to take the Square of one from the Square of the other.

Prob. 15. To find the Ratio of two like Figures, find a third Proportional Aq, Mq : : A, E.

Prob. 16. A Figure being given, to make a like Figure in given Ratio, find a mean Proportional R. √RS : : A. M, if the Ratio of the Figures be as R. S.

Prob. 17. To inſcribe a regular Hexagon in a given Circle, 15 *è* 4.

Prob. 18. To inſcribe a regular Decagon in a given Circle. Let the Semidiameter of the Circle be cut in extreme and mean Proportion, as the 11 *è* 2 directs.

Prob. 19. To inſcribe a regular Pentagon in a given Circle, find the Hypotenuſe of a Rectangular Triangle, whoſe Baſe is the ſide of an Hexagon, and the Perpendicular the ſide of Decagon.

NOTES.

NOTES.

4. The Sum of three Quantities in continual, *&c.*] $Q : \frac{1}{2}A + E = \frac{1}{4}AA + AE + EE$, *or which is all one*, $Q.\ \frac{2}{2}A + E = \frac{4}{4}AA\ (AA) + AE + EE - \frac{3}{4}AA$. *Then* $\frac{1}{2}A + E = \surd u : AA + AE + EE - \frac{3}{4}AA :$

$E = \surd u : AA + AE + EE - \frac{1}{2}A$. *After the ſame manner there will be found* $A = \surd u : AA +$, *&c.*

11. $Q : A - 2N : + Q : 2M - E : = Aq + Eq.$] *It appears thus*,

$$\begin{matrix} A = \surd cAc. \\ E = \surd cEc. \end{matrix} \quad \text{And} \quad \begin{matrix} M = \surd cAqE \\ N = \surd cAEq \end{matrix} \quad \textit{Then}$$

$A - 2N = \surd cAc - 2\surd cAEq.$ *And*

$M - E = 2\surd cAqE - \surd cEc.$ *So that*

$Q : A - 2N : = \surd cAcc - 4\surd cAqqEq + 4\surd cAqEqq.$

$Q.\ 2M - E \cdot = \surd cEcc + 4\surd cAqqEq - 4\surd c\,AqEqq.$ *Wherefore*

$Q : A - 2N + Q : 2M - E : = Aq + Eq.$

16. If in a Circle, *&c.*]

$$\begin{matrix} \delta.\ \pi :: & 2R. & P. \\ & \times\frac{1}{2}R & \times\frac{1}{2}R \end{matrix}$$

$$\begin{matrix} \delta.\ \pi :: & RR. & \frac{1}{2}RP. \\ & \times 2R & \times 2R \end{matrix} \quad \textit{i. e. the Circle.}$$

$\delta . \pi :: 2RRR. RRP.$ *i. e. the Cylinder.*

$\delta . \pi :: \frac{4}{3}RRR. \frac{2}{3}RRP.$ *i. e. the Sphere ; becauſe the Sphere is* $\frac{2}{3}$ *of the Cylinder.*

$\delta . \pi :: \frac{2}{3}RRR. \frac{1}{3}RRP.$ *i. e. the Cone, becauſe the Cone is* $\frac{1}{2}$ *the Sphere ; and becauſe inſerting the firſt Proportion.*

$\pi . \delta (:: P. 2R) :: \frac{1}{2}P. R.$

$\times\frac{1}{2}P \times\frac{1}{2}P.$

$\pi . \delta :: \frac{1}{4}PP. \frac{1}{2}RP.$ *the Circle*

$\times\pi \times\delta \qquad \times P \quad \times R$

$\pi\pi . \delta\delta :: \frac{1}{4}PPP. RRP.$ *the Cylind.*

$\pi\pi . \delta\delta :: \frac{1}{6}PPP. \frac{2}{3}RRP.$ *the Sphere.*

$\pi\pi . \delta\delta :: \frac{1}{12}PPP. \frac{1}{3}RRP.$ *the Cone.*

Theor. 20.] *Here you have theſe Proportions.*

$IA. BA :: FH. BH$

$IA. AC :: CF. FH.$ *Therefore*

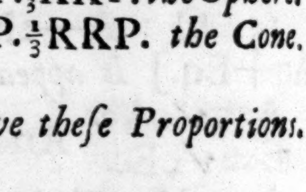
$IA \times BF = BA \times FH.$ *And*

$IA \times FH = AC \times CF.$

Multip.'em, $IAq \times BF \times FH = AC \times BA \times FH \times CF$

Divide by $FH. IAq \times BF = AC \times BA \times CF.$

Multip. by $BF. IAq \times BFq = AC \times BA \times CF \times BF.$



Prob. 18.] *This depends upon the* 9*th. Prop. of the* 13*th Book of* Euclide, *which ſaith, that if you joyn* BE, *the ſide of an Hexagon to* AB *the ſide of a Decagon, (as in the Figure) that* AE *is to* BE :: BE.AB.

Hence it follows, that if you divide the Semidiameter in that which is called extreme and mean Proportion, the greater Segment will be the ſide of the Decagon.

For let AB *be* $=a$
BE $=b$

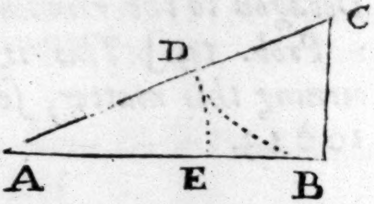
Becauſe $a+b.\ b :: b.\ a.$ (*as is ſuppoſed*)
Then $aa+ab=bb.$
And $aa=bb-ab.$ *Therefore alſo* b *is to* $a :: a.\ b+a.$ *That is, the Radius is to the ſide of the Decagon* : : *as the ſide of the Decagon to the remainder.*

Prob. 19.] *This is that* 11 è 4. *But concerning this matter, ſee* Barrow's *Euclide, Scol* 10 è 13.

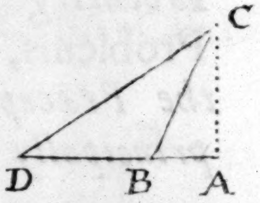
CHAP. XIX.

Examples of Analytical Equation, *for Inventing* Theorems, *and Solving* Problems, *at which Mark* (*as it were*) *the Præcepts hitherto delivered, do principally aim.*

PRop. 1. The Invention of the 11 è 2, *viz:* cut B, a right Line given, ſo that the Rectangle under the whole B and the leſſer Segment, may be equal to the Square of the greater Segment. Call the greater Segment A, the leſſer will be B—A ; draw B—A upon B, and there will be $Bq - BA = Aq$, or $Aq + BA = Bq$. Wherefore $\surd u{:}Bq + \frac{1}{4}Bq{:} - \frac{1}{2}B = A$. By the 9, *Chap.* 16. which *Theorem* is expreſſed in words thus. If to the Square of the Line given, be added a quarter of the ſaid Square, and from the Square-Root of the Sum be taken, half the Line given, the remainder will be the greater Segment.



It is Geometrically effected thus ; Draw $AB = B$, and to it at right Angles ſet $BC = \frac{1}{2}B$: and draw the Hypotenuſe Ac. Thus is $AC = \surd u : Bq + \frac{1}{4}Bq$: Cut off $CD = BC$, the remainder ſhall be $AD = \surd u : Bq + \frac{1}{4}Bq : - \frac{1}{2}B$. Laſtly, meaſure $AE = AD$ for the greater Segment.



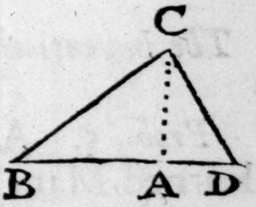
Prob. 2. The Invention of the 12 è 2, *viz.* the comparing the Baſe of an obtuſe Angle with the Sides. Let BCD be a Triangle, whoſe inward Angle at B, is obtuſe. The Baſe is DC, and the Sides BD. BC. Here

$BCq - BAq = CAq = DCq$
$(-DAq$ by the 4 è 2$) - BDq - 2BD \times BA$
$- BAq$. Wherefore $BCq + BDq$ DC
$= BCq - 2BD \times DA$.

In words at length thus. In Obtuſe angled Triangles, the Square of the ſide ſubtending the obtuſe Angle, exceedeth the Sum of the Squares of the Sides, comprehending the ſame Angle by a double Rectangle under one of the Sides about the obtuſe Angle, and the Segment of the ſaid Side (produced) between the obtuſe Angle and the Perpendicular.

Prob. 3. The Invention of the 13 *è* 2. *viz.* the comparing of the Baſe of an acute Angle with the Sides.

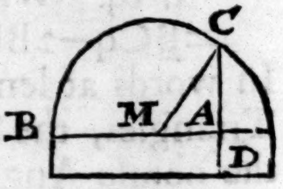
Let BCD be a Triangle, whoſe inward Angle at B is ſuppoſed ſharp. The Baſe is DC, and the Sides BC.BD.



Here $BCq - BAq = CAq = DCq (-DAq$, by the 7th of the 2d.$) - BDq + 2BD \times BA - BAq$. Wherefore $BCq + BDq = DCq + 2BD \times BA$. (After the ſame manner would the Demonſtration proceed; if D did lie between B and A.) In words thus: In acute angled Triangles, the Square of the Side ſubtending, the acute Angle is leſs than the

Sum of the Squares, *&c.* ($2BD \times BA - BAq + DAq = BDq$ ſeventh of the ſecond).

Prob. 4. The Invention of the 14 è 2, *viz.* of a Square equal to the Rectangle $AB \times AD$. Say $AB + AD = 2BM$. Wherefore $AB + AD$ is cut into equal parts at M, and unequal at A. Therefore by the fifth of the ſecond $AB \times AD = BMq - AMq$. Now put $ACq = AB \times AD$, and make a Rectangled Triangle MAC, whoſe Hypotenuſe $CM = BM$ the Semi-Sum of the Sides, and the Baſe AM, equal to the Semi-Difference of the Sides, the Cathetus ſhall be AC, the Root of the Square ſought by 48 è 1.



The Invention of the Area *of a plain Triangle.*

Prob. 5. A certain Friend of mine, a learned Man, brought to me a *Theorem* concerning the Area of a plain Triangle, deſiring me to examin and demonſtrate it. The *Theorem* was, as I remember, (for 'tis many years ago ſince) almoſt in this form, tho not in the ſame Letters.

$$\text{In a plain Triangle, whoſe ſides are A. E. B.}\left\{\begin{array}{l} \frac{1}{8}BqEq - \frac{1}{16}Eqq \\ \frac{1}{8}EqAq - \frac{1}{16}Aqq \\ 1\,AqBq - \frac{1}{16}Bqq \end{array}\right\}\text{Are equal to the Square of the Area of the Triangle}$$

After I had a while conſidered of it there came to my mind 17 è. *Chap.* 18. *Theor.* 20. which ſeemed moſt fit for the Solution of this difficulty. For if the two Legs of a Triangle be A and E, and the Baſe B. From thence it will appear, that $\frac{1}{2}A+\frac{1}{2}E+\frac{1}{2}B$ into $\frac{1}{2}A+\frac{1}{2}E-\frac{1}{2}B$ into $\frac{1}{1}B+\frac{1}{2}A-\frac{1}{2}E$ into $\frac{1}{2}B-\frac{1}{2}A+\frac{1}{2}E$, are equal to the Square of the Area of the Triangle. Multiplying therefore theſe four Magnitudes continually, there will ariſe $\frac{1}{8}AqEq+\frac{1}{8}ApBq+\frac{1}{8}EqBq-\frac{1}{16}Aqq-\frac{1}{16}Eqq-\frac{1}{16}Bqq$. Which is the very Theorem propounded.

And from hence I not only ſatisfied the requeſt, but alſo afforded four other Theorems, more eaſie to be wrought

For becauſe $\frac{1}{2}A+\frac{1}{2}E+\frac{1}{2}B=\frac{1}{2}Z+\frac{1}{2}B$,
and $\frac{1}{2}A+\frac{1}{2}E-\frac{1}{2}B=\frac{1}{2}Z-\frac{1}{2}B$.
And becauſe $\frac{1}{2}B+\frac{1}{2}A-\frac{1}{2}E=\frac{1}{2}B+\frac{1}{2}X$,
and $\frac{1}{2}B-\frac{1}{2}A+\frac{1}{2}E=\frac{1}{2}B-\frac{1}{2}X$.
Therefore $\frac{1}{2}Z+\frac{1}{1}B\times\frac{1}{2}Z-\frac{1}{2}B=\frac{1}{4}Zq-\frac{2}{2}Bq$
and $\frac{1}{2}B+\frac{1}{4}X\times\frac{1}{2}B-\frac{1}{2}X={}_{4}Bq-\frac{1}{2}Xq$

'Tis plain then, That $\frac{1}{4}Zq-\frac{1}{4}Bq$ upon $\frac{1}{4}Bq-\frac{1}{4}Xq=$ to the Square of the Area of the Triangle. In Words thus. If a Quarter of the Difference of the Squares of the Sum of the Legs and the Baſe, be drawn into a quarter of the Difference of the Squares of the Baſe, and the Difference of the Legs, the Product will be equal to the Square of the Area of the Triangle.

Moreover, becauſe $\frac{1}{4}Zq - \frac{1}{4}Bq$ upon $\frac{1}{4}Bq - \frac{1}{4}Xq = \frac{1}{16}Zq + \frac{1}{16}BqXq - \frac{1}{16}Bqq - \frac{1}{16}ZqXq$. Hence in the ſecond place, $Zq + Xq - Bq$ upon $\frac{1}{16}Bq$ mi. $\frac{1}{16}ZqXq =$ Square of the Area of the Triangle.

Alſo becauſe $Zq + Xq = 2Z$, by 11. *Chap.* 18. Hence, Thirdly, $2Z - Bq$ upon $\frac{1}{16}Bq$ mi. $\frac{1}{4}Xq =$ to the Square of the Area of the Triangle. Laſtly, from theſe compared, there will be in the fourth place,

$$\frac{2ZBq - Rqq - Xq}{16} = \text{ to the Square of the}$$

Area of the Triangle. Theſe laſt Theorems are eaſie to be expreſſed in Words.

Prob. 4. The Solution of the Problems concerning Arithmetical Progreſſion into Propoſitions. Let the Symbols be theſe; α, the firſt and leaſt Term; ω, the laſt and greateſt; T, the Number of Terms; X, the Common Difference; Z, the Sum of all the Terms. Therefore $T - 1$ is the Number of Differences. Hence $TX - X = \omega - \alpha$ the Sum of the Differences. Three of theſe five, α, ω, T, X, Z. To find the reſt by the 20 following Propoſitions (for ſo many are the varieties) in this order.

Given.

Given.	Sought.	By Prop.
α, ω, T,	Z & X	1 & 2
α, ω, X,	T & Z	3 & 4
α, ω, Z,	T & X	5 & 6
α, T, X,	ω & Z	7 & 8
α, T, Z,	ω & X	9 & 10
α, X, Z,	ω & T	11 & 12
ω, T, X,	α & Z	13 & 14
ω, T, Z,	α & X	15 & 16
ω, X, Z	α & T	17 & 18
T, X, Z,	x & ω	19 & 20

Prop. I. $T\omega + T\alpha = 2Z.$

II. $\dfrac{\omega - \alpha}{T - 1} = X.$

III. $\dfrac{\omega - \alpha}{X} + 1 = T$ by 2.

IV. $\dfrac{\omega q - \alpha q}{X} + \omega + x = 2\,Z$ by 1. 3.

V. $\frac{2Z}{\omega+\alpha} = T$ by 1.

VI. $\frac{\omega q - \alpha q}{2Z - \omega - \alpha} = X$ by 4.

VII. $TX - X + \alpha = \omega$ by 2.

VIII. $TX - X + 2\alpha$ upon $T = 2Z$ by 1 and 7.

IX. $\frac{2Z - T\alpha}{T} = \omega$ by 1.

X. $\frac{2Z - 2\alpha}{Tq - T} = X$ by 8.

XI. $\sqrt{u}:\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX: - \frac{1}{2}X = \omega$. by 4.

XII. $\sqrt{u}:\frac{\alpha q - \alpha X + \frac{1}{4}Xq + 2ZX}{Xq}: - \alpha + \frac{1}{2}X = T$ by 8.

XIII. $\omega + X - TX = \alpha$ by 7.

XIV. $2\omega + X - TX$ upon $T = 2Z$ by 1 & 13.

XV. $\frac{2Z}{T} - \omega = \alpha$ by 9.

XVI. $\frac{2T\omega - 2Z}{Tq - T} = X$ by 14.

XVII. $\frac{1}{2}X \pm \sqrt{u}: \frac{\omega q + \omega X + \frac{1}{4}Xq - 2ZX}{Xq} : = \alpha$ by 4

according as α ſhall be $\left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\}$ than $\frac{1}{2}X$.

XVIII. $\frac{\omega + \frac{1}{2}X}{X} \mp \sqrt{u}: \frac{\omega q + \omega X + \frac{1}{4}Xq - 2ZX}{Xq}$

$= T$ by 14.

according as α ſhall be $\left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\}$ than $\frac{1}{2}X$.

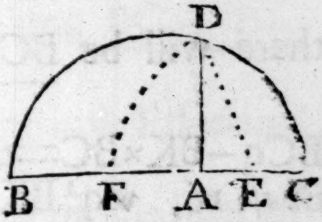
XIX. $\frac{2Z}{2T} - \frac{TX}{2} + \frac{X}{2} = \alpha$ by 10.

XX. $\frac{2Z}{T} + \frac{TX}{2} - \frac{X}{2} = \omega$ by 16.

Prob. VII. *Euclid* 11. *è* 2. hath taught us to cut a given Line ſo, that the Rectangle under the whole, and the leſſer Segment ſhall be equal to the Square of the greater Segment; which Section is almoſt divine. Now let that Problem be propoſed generally. Let the given line AB be ſo cut, that the Rectangle under the whole AB and leſſer Segment may bear any ratio whatſoever to the Square of the greater Segment. Let it be as R to S:

Firſt ſay R. S :: AB. AC. which may be a fourth Proportional: then for the greater Segment put A: the leſſer Segment will be $AB - A$: which being drawn into AB will give the Rectangle $ABq - AB \times A$. There will be then AB. AC :: $ABq - AB \times A$. Aq. and therefore by 3. *Chap.* 6. $ABq \times AC - AB \times AC \times A = AB \times Aq$, and by a Diviſion them by of AB. there will be $AB \times AC - AC \times A = Aq$ or $Aq + AC \times A = AB \times AC$, and by 9. *Chap.* 16. there is found $\surd u{:}\ \frac{1}{4} ACq + AB \times AC{:} - \frac{1}{2} AC = A$.

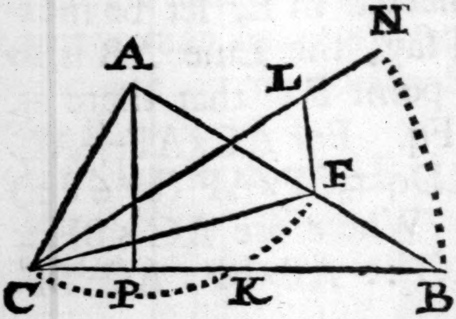
This Theorem is expreſſed in Words thus. If to the Square of half the fourth Proportional be added a Rectangle under the given Line and that fourth Proportional; and out of the ſquare Root of the Sum be taken half the fourth Proportional, the Remainder will be the greater Segment. Geometrically thus, let AB and AC be joined in a direct Line, and to the Diameter BC draw a Semicircle, and upon BC in the point A erect a perpendicular AD, cutting the Semicircle in D. Then AC being biſected in E, let be meaſured EF=ED. I ſay, the Line AB is ſo ſo divided in the point F, that there is, R. S :: AB×BF . AFq. For AC×AF+AC ×BF=AC×AB=ADq=CF×AF (by 6 è 2) =AC×AF+AFq. Wherefore AC×BF= AFq. But AB . AC :: AB×BF . AC×BF. therefore, *&c.*



Prob. VIII. Either ſide of a rectangular Triangle (in which a Perpendicular from the right angle cuts the Hypotenuſe) being given, together with BK the difference of the Segments of the Hypotenuſe: to find both the Hypotenuſe and the Triangle it ſelf. Firſt let the leſſer Side CA be given, ſuppoſe it be done, and let the rectangular Triangle be BAC. in which from the Vertex let fall upon the Hypotenuſe the Perpendicular AP, cutting the Hypotenuſe into the the Segments BP and CP. But $CP=\frac{BC-BK}{2}$.

And becauſe $BC . CA :: CA . \frac{BC - BK}{2}$ there will be $\frac{BCq - BC \times BK}{2} = CAq$. or $BCq - BK \times BC = 2CAq$. Wherefore by 9. *Chap.* 16, $\surd q \tfrac{1}{4}BKq + 2CAq : + \tfrac{1}{2}BK = BC$.

This Theorem is expreſſed in Words thus,



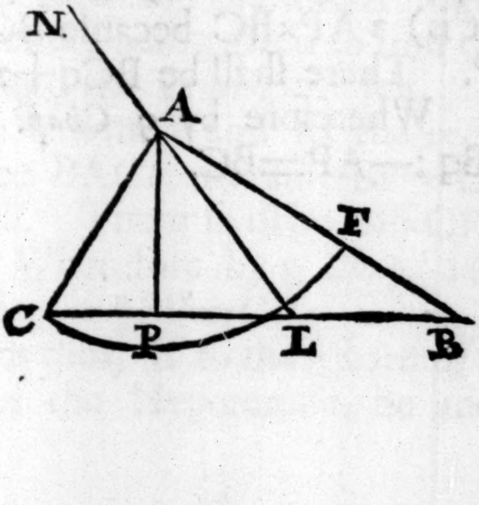
If the Square of half the difference of the Segments of the Hypotenuſe be added to 2 Squares of the given ſide; and the Square root of the aggregate be increaſed with half the difference, the Product will be equal to the Hypotenuſe.

Geometrically thus, there being taken $AF = AC$, let there be drawn CF, and perpendicular to it $FL = \frac{BK}{2}$, and let CL be produced to N, that $LN = \tfrac{1}{2}BK$; there will be $CN = BC$. Wherefore let there be inſcribed in a Circle $CK = CN - BK$ and produce *&c.* For $CFq = 2CAq$, and $CLq = 2CAq + \tfrac{1}{4}BKq$, therefore, *&c.*

But if the greater ſide BA be given, ſuch an Equation ſhall be found, $\sqrt{q} : \frac{1}{4}BKq + 2BAq : -\frac{1}{2}BK = BC$. Taking $\frac{BC + BK}{2}$ for PB. And the Geometrical Effection not unlike the former.

Prob. IX. BF the difference of the ſides of a rectangled Triangle, and AP, a Perpendicular from the right Angle to the Hypotenuſe being given, to find the Hypotenuſe and the Triangle it ſelf.

Suppoſe it done: and let the rectangled Triangle be BAC. Becauſe by 7 è 2 $2BA \times AF + BFq = BAq + AFq$; and therefore $BFq =$ ($ABq + AFq$, that is) $BCq -$ ($BA \times 2CA$, that is) $BC \times 2AP$, becauſe $BC . CA :: BA . AP$. There will be $BCq - 2AP \times BC = BFq$. Wherefore by 9 *Chap* 16, $\sqrt{u} : APq + BFq : + AP = BC$.

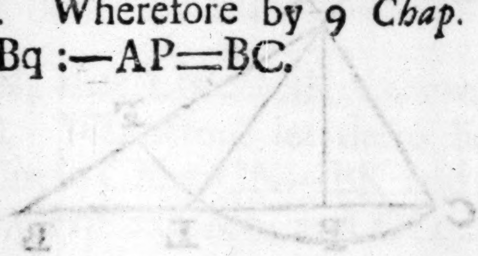


This Theorem is expreſſed in Words thus. If to the Square of the Perpendicular be added the the Square of the difference of the ſides, and the ſquare-root of the aggregate be increaſed with the Perpendicular. The Product ſhall be equal to the Hypotenuſe.

Geometrically thus. make PL=BF. And let LA be extended to N, ſo that AN=AP. There ſhall be LN=BC. Therefore on the Diameter BC deſcribe a Semicircle, in which erect a Perpendicular equal to AP given. Then draw BA and CA.

Prob. X. BG the Sum of the ſides of a rectangled Triangle and AP a perpendicular from the right Angle to the Hypotenuſe being given: to find the Hypotenuſe and the Triangle it ſelf.

Suppoſe it done, and let the rectangled Triangle be BAC. Becauſe by 4 è 2· BGq = (BAq+GAq, that is) BCq+ (2BA×CA, that is) 2 AP×BC becauſe BC . CA :: BA . AP. There ſhall be BCq+2AP×BC = BGq. Wherefore by 9 *Chap*. 16, √q: APq+Bq :—AP=BC.



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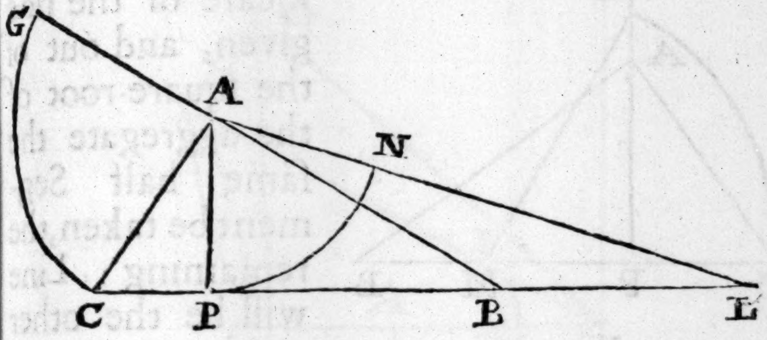


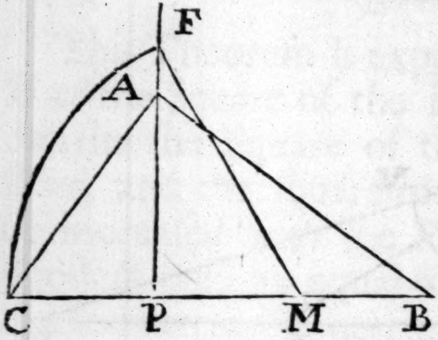
In Words thus, If to the ſquare of the perpendicular be added the ſquare of the ſum of the ſides, and out of the ſquare-root, of the aggregate the perpendicular be deducted, the remaining Line ſhall be equal to the Hypotenuſe.

Geometrically thus, make $PL = Bq$. and draw AL from which cut off $AN = AP$. There will be $LN = BC$. On the Diameter BC deſcribe a Semi-circle, *&c.*

Prob. XI. CA one ſide of a rectangled Triangle, and BP the altern Segment of the Hypotenuſe being given, to find the other Segment and the Triangle it ſelf.

Suppoſe it done: and let the rectangled Triangle be BAC. Becauſe $BP + CP . CA :: CA . CP$. There ſhall be $BP \times CP + CPq = CAq$. Wherefore by 9 *Chap.* 16, $\sqrt{q}\,\frac{1}{4} BPq + CAq : - \frac{1}{2} BP = CP$.

In Words thus, If to the ſquare of half the Segment of the Hypotenuſe, be added the ſquare

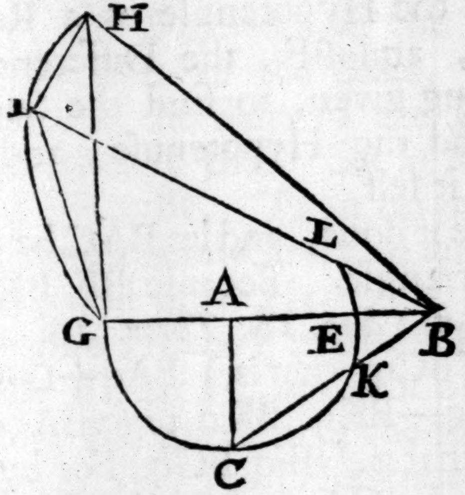


ſquare of the ſide given, and out of the ſquare-root of the aggregate the ſame half Segment be taken, the remaining Line will be the other Segment of the Hypotenuſe.

Geometrically thus, ſet at right Angles BP and PF=CA, and biſecting BP at M, draw MF, and meaſure MC=MF. Therefore the other Segment CP is found and likewiſe BC the whole Hypotenuſe. On the Diameter BC deſcribe a ſemi-circle, in which inſcribe CA and BA.

Prob. XII. BK the Difference of the Segments of the Hypotenuſe of a rectangled Triangle and BG the ſumm of the ſides being given, to find both the difference of the ſides and the Hypotenuſe and alſo the Triangle it ſelf.

Suppoſe it done, and let the rectangled Triangle be BAC. Becauſe BG . BK :: BC. BF. by 17 *Chap.* 18, *Theor.* 16. And BGq. BKq :: (BCq that is) BAq+CAq. BFq. Alſo 2BGq−BKq. BKq :: (2BAq+2CAq, BFq that is) BGq. BFq. for by 8 *Chap.* 18, 2BAq+2CAq=BGq+BFq. Wherefore √q: 2BGq−BKq.BG :: BK. BF :: BG. BC. In

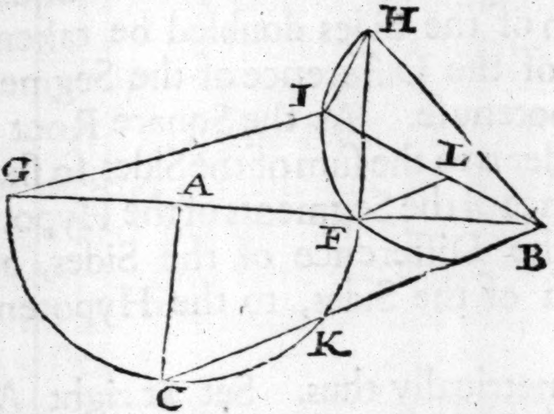


In Words thus. If from the Square of the Sum of the Sides doubled be taken, the Square of the Difference of the Segments of the Hypotenuſe. As the Square Root of the remainder is to the ſum of the Sides, ſo ſhall the Difference of the Segments of the Hypotenuſe be, to the Difference of the Sides, and ſo the Sum of the Sides, to the Hypotenuſe.

Geometrically thus. Set at right Angles BG and GH$=$BG. Then on the Diameter BH deſcribe a Semi-Circle; in which inſcribe HI$=$BK, and draw BI. Therefore BI$=\surd$q: 2BGq—BKq. Make alſo BL$=$BK, draw GI, and to it a Parallel LF. Therefore BF the Difference of the Sides is found.

Prob. 13. BK the Difference of the Segments of the Hypotenuſe of a Rectangled Triangle, and BF, the Difference of the ſides, being given, to find the Sum of the Sides, and the Hypotenuſe, and alſo the Triangle it ſelf.

Suppoſe it done, and let BAC be the Rectangled Triangle. Becauſe BF. BK : : BC. BG. By 17. *Chap.* 18. *Theor.* 16. and BFq. BKq : : (BCq, that is,) BAq+CAq. BGq. Alſo 2BFq—BKq. BKq : : (2BAq+2CAq—BGq, that is,) BFq. BGq. For by 8. *Chap.* 18. 2BAq+2CAq=BGq+BFq.



Wherefore √q2BFq—BKq. BF : : BK. BG : : BF. BC. In Words thus. If from the Square of the Difference of the Side doubled be taken, the Square of the Difference of the Segments of the Hypotenuſe, as the Square-Root of the Remainder is to the

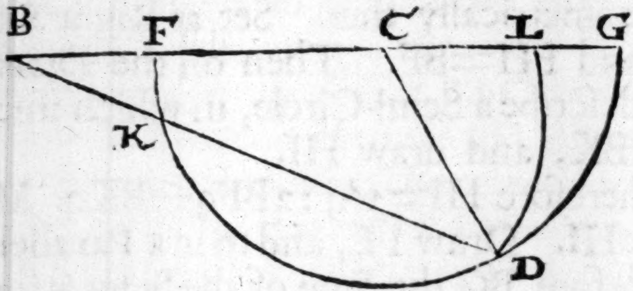
Difference of the Sides, ſo ſhall the Difference of the Segments of the Hypotenuſe be, to the Sum of the Sides, and ſo the Difference of the Sides to the Hypotenuſe.

Geometrically thus. Set at Right-Angles BF and FH=BF. Then on the Diameter BH deſcribe a Semi-Circle, in which inſcribe BI=BK, and draw HI.

Therefore $HI=\sqrt{q}:2BFq-BKq$. Make BL=HI. Draw FL, and to it a Parallel IG. Therefore BG the Sum of the Sides is found.

Prob. 14. BF, the Difference of the Sides of any plain Triangle, BK the Difference of the Segments of the Baſe, and CL, the Difference between the greater Side, and the Baſe being given; to find the Baſe, the Sum of the Sides, and the Triangle it ſelf. And firſt, let the Exceſs be in the Baſe: ſuppoſe it done, and let BCD be the Triangle.

Becauſe FB. BK : : BD. $\frac{BK\times BD}{BF}=BG$, by 17, *Chap.* 18. *Theor.* 16. There ſhall be $\frac{BK\times BD-BFq}{2BF}=CF$. Add BF, and $\frac{BK\times BD+BFq}{2BF}=BC$. Take this out of BD, & $\frac{2BF\times BD-BK\times BD-BFq}{2BF}=\frac{2BF\times CL}{2BF}$. Wherefore $2BF \times BD - BK \times BD = 2BF \times CL + BFq$. And by 3: *Chap*. 6. $2BF - BK.\ 2CL + BF :: BF.\ BD :: BK.\ BG$.



In Words thus. As the Difference between the doubled Difference of the Sides, and the Difference of the Segments of the Baſe, is to the Aggregate of the double Difference, between the greater Side and the Baſe, and the Difference of the Sides; ſo the Difference of the Sides is to the Baſe : And ſo the Difference of the Segments of the Baſe, to the Sum of the Sides.

The Geometrical Praxis is ſo eaſie, that it may be omitted.

But if the Exceſs be in the greater Side, the *Theorem* will be $BK - 2BF.\ 2CL - BF :: BF.\ BD :: BK.\ BG$. I omit the Inveſtigation of this Theorem, and the Solution of the Problem, wherein, out of BG, the Sum of the Sides of any plain Triangle, and BK the Difference of the Segments of the

Baſe, and CL the Difference between the greater Side, and the Baſe being given; It is required to find the Baſe, and the Difference of the Sides, that the Reader may have wherewithall to exerciſe his skill.

Prob. 15. BG the Sum of the Sides of any plain Triangle. BK the Difference of the Segments of the Baſe, and CA a Perpendicular being given, to find out both the Baſe, and the Difference of the Sides, and the Triangle it ſelf.

Suppoſe it done, and let BCD be the Triangle; becauſe by 17, *Chap.* 18. *Theor.* 16. $BG.\ BD :: BK.\ BF$. And by 5, *Chap* 18. $DKq = BDq + BKq - 2BK \times BD$. And by 47 è 1. ($4ADq$, that is,) $DKq + 4CAq =$ ($4CDq$, that is,) FGq. There will be

$$BDq + BKq - 2BK \times BD + 4CAq = FGq.$$

Take FG out of BG: and $BG - \sqrt{q} : BDq + BKq - 2BK \times BD + 4CAq : = BF$.

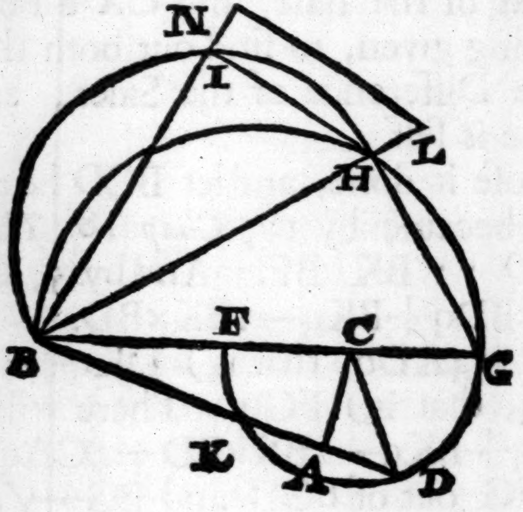
Wherefore there will be

$$BG.\ BD :: BK.\ BG - \sqrt{q} : BDq + BKq - 2BK \times BD + 4CAq.$$

And by 3, *Chap.* 6. $BK \times BD = BGq - \sqrt{q} : BGq \times BDq + BGq \times BKq - BGq \times 2BK \times BD + BGq \times 4CAq$. Therefore by 8, *Chap.* 16. $Q : BGq - BK \times BD$; that is, $BGqq - \underline{BGq \times 2BK \times BD} + BKq \times BDq = BGq \times BDq + BGq \times BKq - \underline{BGq \times 2BK \times BD} + BGq \times 4CAq$.

And therefore $BGq \times BDq - BKq \times BDq = BGqq - BGq \times BKq - BGq \times 4CAq$. Or alſo $BGq - BKq$ upon $BDq = BGq - BKq - 4CAq$ upon BGq.

Therefore $\sqrt{q} : BGq - BKq \quad \sqrt{q} : BGq - BKq - 4CAq :: BG . BD :: BK . BF$.



In Words thus. As the Square Root of the Difference between the Squares of the Sum of the Sides, and the Difference of the Segments of the Baſe, is to the Square Root of the ſame Difference, leſs by the Square of the Perpendicular doubled; ſo is the Sum of the Sides to the Baſe; and ſo the Difference of the Segments of the Baſe, to the Difference of the Sides.

Geometrically thus. On BG the Diameter, deſcribe a Semi-Circle; in which inſcribe $GH=BK$, and BH. Therefore $BH=\surd q: BGq-BKq$. Again, on the Diameter BH draw a Semi-Circle, in which inſcribe $HI=2CA$: and BI. Therefore $BI=\surd q BGq-BKq-4CAq$. Make $BL=BG$: and from L draw LN, Parallel to HI, concurring with BI, produced in the Point N: Here is found therefore $BN=BD$.

Prob. 16. BF the Difference of the Sides of any plain Triangle; BK the Difference of the Segments of the Baſe; and CA a Perpendicular being given: To find both the Baſe, and the Sum of the Sides, and alſo the Triangle it ſelf.

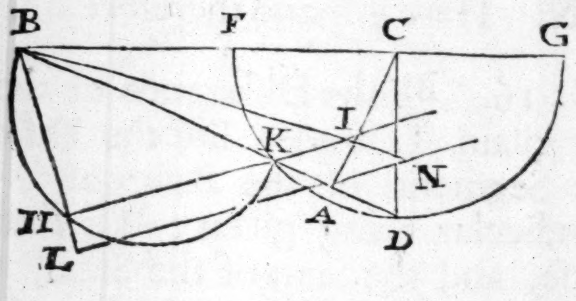
Suppoſe it done. Let BCD be the Triangle.

$BG . BD :: BK . BF$. And $DKq=BDq+BKq-2BK\times BD$, by 5, *Chap.* 18. and by 47 è 1, ($4ADq$, that is,) $DKq+4CAq=$ ($4CDq$, that is,) FGq.

There ſhall be $BDq+BKq-2BK\times BD+4CAq=FGq$. Add FG to BF: and $BF+\surd q: BDq+BKq-2BK\times BD+4CAq: =BG$.

Wherefore $BF . BD :: BK . BF+\surd q: BDq+BKq-2BK\times BD+4CAq$. Alſo $BK\times BD=BFq+\surd q: BFq\times BDq+BFq\times BKq-BFq\times 2BK\times BD+BFq\times 4CAq$. Therefore $Q: BK\times BD-BFq$; that is, $BKq\times BDq-BFq\times 2BK\times BD+BFqq=BFq\times BDq+BF$

$q \times BKq - \underline{BFq \times 2BK \times BD} + BFq \times 4CAq$, And therefore $BKq \times BDq - BFq \times BDq = BFq \times BKq - BFqq + BFq \times 4CAq$, or alſo $BKq - BFq$ upon $BDq = BKq - BFq + 4CAq$ upon BFq. Therefore $\surd q : BKq - BFq.\ \surd q : BKq - BFq + 4CAq :: BF.\ BD :: BK.\ BG.$

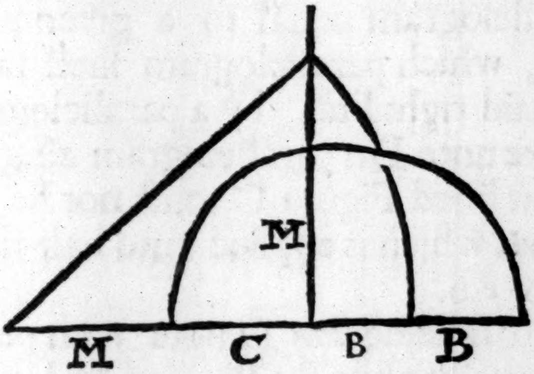


In Words thus, As the ſquare-root of the difference between the ſquares of the difference of the Segments of the baſe, and of the difference of the ſides, is to the ſquare-root of the ſame difference increaſed with the ſquare of the double perpendicular, ſo is the difference of the ſides, to the baſe; and ſo the difference of the ſegments of the baſe to the ſumm of the ſides.

Geometrically thus, on BK the Diam. deſcribe a ſemicircle in which inſcribe $KH = BF$ and BH. Therefore $BH = \surd q : BKq - BFq$. make $BHL = BF$ and $HKI = 2CA$. draw BI. Therefore $BI = \surd q : BKq - BFq + 4CAq$. draw alſo LN parallel to HI concurring with BI extended in the point N. here is therefore found $BN = BD$.

Prob. XVII. In a rectangled Triangle, B the difference between the Baſe and the Hypotenuſe, and C the difference between the Cathetus and the Hypotenuſe being given; to find both the Hypotenuſe, and the Triangle it ſelf.

For the Hypotenuſe put A, the Baſe will be $A - B$, and the Cathetus $A - C$, and by 47 è 1, the Cathetus is $\surd q: 2BA - Bq$. Wherefore $\surd q: 2BA - Bq = A - C$. and $2BA - Bq = Aq - 2CA + Cq$. or $2B + 2C$ upon A mi $Aq = Bq + Cq$. Therefore by 9 *Chap.* 16, $B + C + \surd q: 2BC = A$ the Hypotenuſe.



In Words thus. The Aggregate of the two differences, that is, the difference both of the Baſe and of the Cathetus from the Hypotenuſe, together with the ſquare-root of a double rectangle under the differences is equal to the Hypotenuſe.

Geometrically thus, draw an infinite line in which meaſure B, B, and C. This being taken for the Diameter, draw a ſemi-circle, and in the common point of B and C, erect perpendicularly the Line M. Therefore $Mq = 2BC$. alſo from C in the infinite line meaſure M, and with the ſemi-diameter $M + C + B$ deſcribe an arch, until it concur with the perpendicular line M produced. Then from the point of concourſe to the centre of that arch draw a line for the Hypotenuſe, and ſo the ſought rectangled Triangle ſhall be deſcribed.

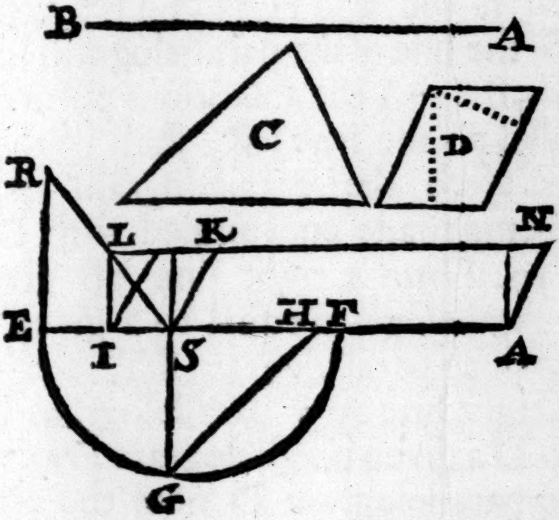
Prob. 18. Unto a given line AB to apply a Parallelogram equal to a given plain Figure C, which parallelogram ſhall fall ſhort of the ſaid right line, by a parallelogram Figure like unto D a parallelogram aſſign'd. But the right lined Figure C muſt not be greater than that which is applied unto half that line. *Prop.* 28. *è* 6.

In a parallelogram D note with perpendicular lines its altitudes R, and its latitude S: it is no matter whether of them be greater.

For the ſide of the parallelogram ſought put A: the ablatitious portion ſhall $AB - A$.

Make $S . R :: AB - A . \frac{AB \times R - R \times A}{S}$: The altitude of the Parallelogram ſought; which being drawn upon A the ſide ſhall give

$\frac{AB \times R \times A - R \times Aq}{S} = C$: or $AB \times A - Aq = \frac{C \times S}{R}$ and by 9 *Chap.* 16. $\frac{AB}{2} + \sqrt{q}: \frac{ABq}{4} - \frac{C \times S}{R} = A$.



In Words thus, If C the plain figure given be drawn into the latitude of the parallelogram D, and the product be divided by the Altitude and the quotient taken out of the ſquare of half the given line AB, the ſquare-root of the remainder encreaſed with the ſame half, ſhall be the ſide of the parallelogram ſought.

Geometrically thus, make ER $= \sqrt{q}C$, Then R. S :: ER. ES. Set ER and ES at right angles, and taking SF $=$ ER, on the Diameter EF deſcribe a ſemi-circle, in which erecting a perpendicular SG, there will be $SGq = \frac{C \times S}{R}$ from the point G meaſ. GH $= \frac{1}{2} AB = HB$ there will be $HS = \sqrt{:} \frac{ABq}{4} - \frac{C \times S}{R}:$ to which if you add $HA = \frac{1}{2} AB$; you ſhall have AS, the ſide of the parallelogram ſought. And $BS = BA - A$ the ablatitious portion, and BL parallel to the line ER, ſhall be the altitude, therefore the parallelogram ſought is ASKN being made equi-angled with D.

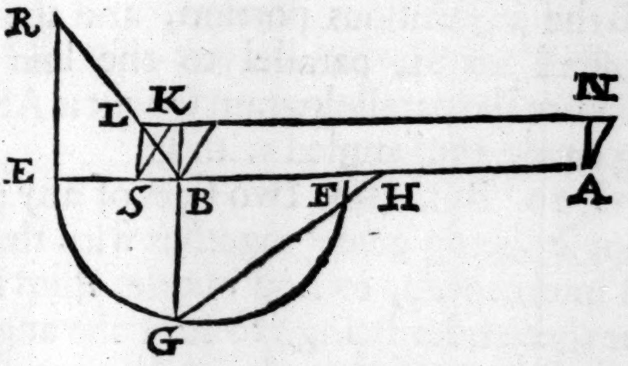
Prob. 19. Unto a right line AB given to apply a parallelogram equal to a given plain figure C, which parallelogram ſhall over-reach the ſaid right line by a parallelogram-figure like unto D a given parallelogram. *Prop.* 29. *è* 6.

In the parallelogram D note the altitude and latitude, as in the former.

For the ſide of the parallelogram ſought put A: the adjectitious portion ſhall be $A - AB$, make $S. R :: A - AB. \frac{R \times A - AB \times R}{S}$ the altitude of the ſought parellelogram: which being drawn on A the ſide ſhall give $\frac{R \times Aq - AB \times R \times A}{S} = C.$ or $Aq - AB \times A = \frac{C \times S}{R}$.

And by 9 *Chap.* 16, $\sqrt{q}: \frac{ABq}{4} + \frac{C \times S}{R} : + \frac{AB}{2} = A.$

In



In Words thus, if C the plain Figure given be drawn into the latitude of the parallelogram D; and the product be divided by the altitude; and the quotient added to the ſquare of the half the given line AB: the ſquare-root of the aggregate increaſed with the ſame half, ſhall be the ſide of the parallelogram ſought.

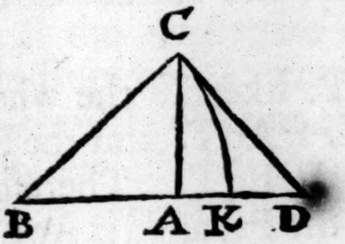
Geometrically thus, make $ER=\surd qC$. Then $R.S :: ER.\ EB=\surd\frac{C\times S}{R}$. Set ER and EB at right angles: and taking $BF=ER$, on the diameter EF deſcribe a ſemi-circle, in which erect a perpendicular BG, and there ſhall be $BGq=\frac{C\times S}{R}$. Meaſure $BH=\frac{1}{2}AB=AH$. and draw $GH=\surd q:\frac{ABq}{4}+\frac{C\times S}{R}:=HS$. Therefore $AS=A$ the ſide of the parallelogram ſought, and BS=A —AB the adjectitious portion, and the altitude ſhall be SL parallel to the line ER. Therefore the parallelogram ſought is ASKN being made equi-angled with D.

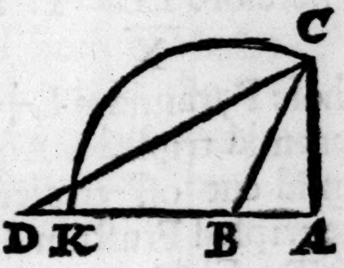
Prob. 20. BC, BD, two ſides of any plain Triangle being given together with the angle B intercepted, to find out the third ſide; or the three ſides being, to find the angle B oppoſite to one of them.

Suppoſe it done, and let BCD be the Triangle. On the center B and ſemi-diameter BC, deſcribe an arch CK, and drop a perpendicular CA. Therefore KD is the difference of the ſides: and AK the like to the verſed ſine of the angle B. For Rad. sv B : : BK. $AK = \frac{BK \times sv\,B}{Rad.}$ but $AK = BK \mp BA$, as appeareth by the Schemes compared.

$$\text{And bec. } BDq + BKq = \text{both} \begin{Bmatrix} 2BD \times KB + KDq. \text{ by 5 } \textit{Chap.} \text{ 18.} \\ CDq \pm 2BD \times BA. \text{ by 2, 3 } \textit{Chap.} \text{ 19.} \end{Bmatrix}$$

There will be $2BD \times BK + KDq = CDq \pm 2BD \times BA$. Wherefore $2BD \times BK \mp 2BD \times BA$, that is, $2BD \times AK + DKq = CDq$. but $2BD \times AK = \frac{2BD \times BC \times sv\,B}{Rad.}$ Therefore $\frac{2BD \times BC \times sv\,B}{Rad.} + DKq = CDq$. which is the firſt Theorem, and $\frac{CDq - KDq \text{ upon Rad:}}{2BD \times BC} = \text{sv } B$. which is the ſecond Theorem.



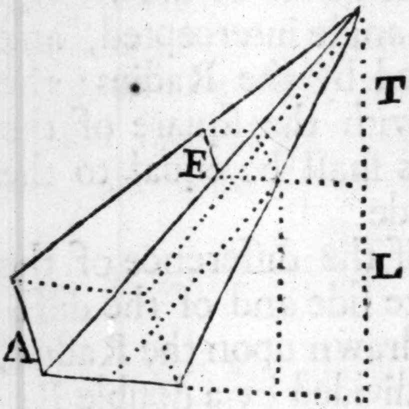


The firſt in Words thus, if a double Rectangle under the ſides given be drawn into the verſed ſine of the angle intercepted, and the product be divided by the Radius: the Quotient increaſed with the ſquare of the difference of the ſides ſhall be equal to the ſquare of the third ſide.

The ſecond thus, if the difference of the ſquares of the oppoſite ſide and of the difference of the ſides be drawn upon the Radius, and the product be divided by a double Rectangle under the containing ſides; the Quotient ſhall be equal to the verſed ſine of the angle ſought.

Prob. 21. In a piece or *Fruſtum* of a Pyramid both the baſes Aq and Eq and the altitude L being given, to find the meaſure of the *Fruſtum*. You muſt firſt know (out of 7 and 10 è 12) that a parallelepipedon is equal to 3 Pyramids, and a Cylinder to three Cones of the ſame baſe and altitude.

The altitude T of the Pyramid cut off muſt firſt be ſought, thus, A—E. E : : L.T. Wherefore $\frac{LE}{X} = T$. and the altitude of the whole Pyramid is L+T. likewiſe the whole Pyramid tripled is AqL+AqT. and the Pyramid cut off tripled is EqT. Therefore the tripled Fruſtum of the Pyramid is AqL+AqT−EqT.



This Theorem ſheweth one way of meaſuring the Fruſtum of a Pyramid. In words thus, if the ſolid under the greater baſe, and the whole altitude be leſſened by a ſolid under the leſſer baſe and altitude of the Pyramid cut off; the third part of the Remainder ſhall be equal to the Fruſtum.

Again becauſe by 2 *Chap.* 11, Aq−Eq= ZX: and $T = \frac{LE}{X}$. there ſhall be AqL+ (ZEL, that is, by 3 è 2) AEL+EqL=AqL +

+AqT—EqT. Therefore the tripled Fruſtum of the Pyramid is **Aq+Eq+AE upon L.**

This Theorem teacheth another way of meaſuring the Fruſtum of a Pyramid, in words thus,

If the aggregate of both the baſes of the Fruſtum of a Pyramid, and of the mean proportional between them be drawn upon the altitude of the Fruſtum, the third part of the product ſhall be equal to the Fruſtum.

Alſo becauſe by 2 *Chap* 11, 2Aq+2Eq= Zq+Xq : ZqL + XqL + 2AEL ſhall be equal to ſix Fruſta's. but by 11 *Chap.* 18, Xq +2AE=Z. therefore Zq+Z upon L is equal to ſix times the Fruſtum of a Pyramid. And this Theorem teacheth a third way of meaſuring the *fruſtum* of a Pyramid, in words thus, If to the aggregate of the baſes be added the ſquare of the aggregate of the ſquare roots of both baſes, and their ſum be drawn upon the altitude of the Fruſtum: the ſixth part of the Product ſhall be equal to the Fruſtum.

But if the queſtion be about meaſuring the Fruſtum of a Cone. Becauſe according to *Archimedes* his diſcovery, the ſemi-circumference of a Circle is equal to $\frac{22}{7}$ of the Radius (or ſemi-diameter) *ferè*: or more accurately $\frac{355}{113}$ Rad. the area of a Circle ſhall be $\frac{355}{113}$ Rad : q. And 113. 355 :: Rad : q. Area of the Circle.

Wherefore the firſt Theorem for meaſuring the Fruſtum of a Cone is, $\frac{355}{452}$ AqL $+$ $\frac{355}{452}$ AqT $-$ $\frac{355}{452}$ EqT are equal to the Fruſtum tripled.

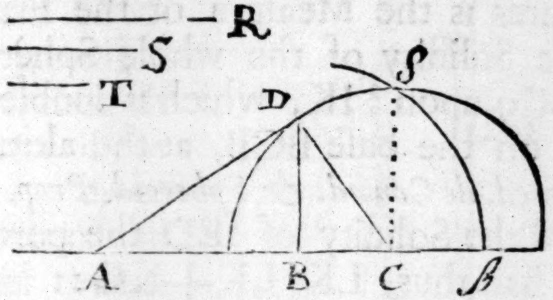
The ſecond is $\frac{355}{452}$ Aq $+$ $\frac{355}{452}$ Eq $+$ $\frac{355}{452}$ AE upon L, are equal to the Fruſtum tripled.

The third is $\frac{355}{452}$ Zq $+$ $\frac{355}{452}$ Z upon L, are equal to the Fruſtum of a Cone ſixfold.

Prob. 22. A Problem of *Apollonius Pergæus* ἐν ἀναλυομένῳ τόπῳ. Two points A, B, being given in a plain, to deſcribe a Circle, to whoſe Circumference the right-lines AB, BD, drawn from the ſame Points, ſhall have the ratio of R to S.

Suppoſe it done, and let the Center of the Circle ſought be C in the ſame right-line with the points A, B; and the ſemi-diameter CD. Make R. S : : S. T. Now becauſe the two Triangles ACD, DCB. (whereſoever the point D be taken) are as AC to BC: and the ſides DA, DB, alike oppoſite to the common Angle C, are in the Ratio of R to S: and the ſide CD common to both: one may eaſily conceive the Triangles ACD, DCB to be like. and by 1 *Chap.* 15, AC. AC : : Rq. Sq : : R. T. If therefore for BC be put A : there ſhall be AB $+$ A. A : : RT. and AB$\times$T $+$ T$\times$A $=$ R$\times$A : or $\frac{AB\times T}{R-T} = A$;

Laſtly, $\sqrt{}$: AC$\times$BC : $=$ DC. In



In Words thus, If the ſpace between the points be drawn into the third proportional Term of the ratio given: and the Product be divided by the exceſs of the firſt Term above the third: the Quotient ſhall be equal to the diſtance of the nearer point from the Center: and the ſquare root of the Rectangle under both diſtances from the Center ſhall be equal to the ſemi-diameter. The Geometrical Effection is eaſie.

Prob. 23. In a Tun or Wine-veſſel, the internal Longitude 2CL; and the two ſemi-diameters, namely, CB at the Bung, and LD at the Head, being given, to find the capacity of the Tun it ſelf. A Tun is a Fruſtum of a Sphæroides, which ſolid Figure is made by the revolution of half an *Ellipſis* (or Oval) about its tranſverſe diameter or *axis*. Now that the meaſure of the Fruſtum may be found out, the Meaſures both of the whole Spheroides and of the Portions cut off, muſt be known. For the difference of theſe

Meaſures is the Meaſure of the Fruſtum.

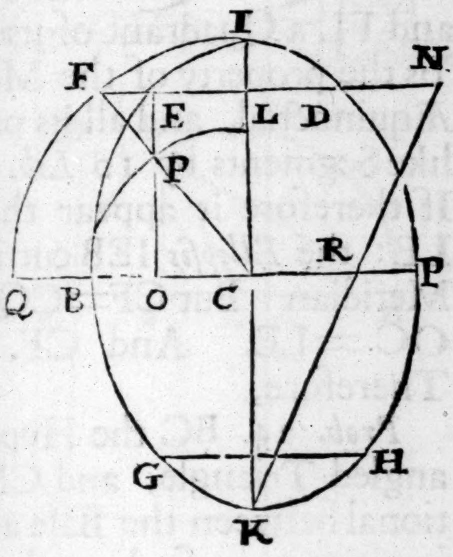
The Solidity of the whole Spheroides is $\frac{355}{113}$ BCq upon $\frac{2}{3}$ IK, which is double a cone made on the baſe BCB, at the altitude IK. *Archimed. de Conoid. & Spheroid. Prop.* 29.

And the Solidity of IED the portion cut off is had thus, LK. LK+KC :: $\frac{355}{113}$ LDq upon $\frac{1}{3}$ LI. Solidity ſought. *Ibid. Prop.* 31.

But there is yet wanting (that which is the main hinge of this buſineſs) the tranſverſe diameter or *Axe* IK: which you ſhall thus find.

Suppoſe that done which is required; deſcribe an *Ellipſis*, *&c.* as in the ſcheme. And make CK. CB :: CB. $\frac{\text{CBq}}{\text{CK}} = \text{CR}$, half the right *latus* by 13 *Lib.* 1 *Conic. Apollon.* Again make CK. $\frac{\text{CBq}}{\text{CK}}$ :: CK+CL.

$\frac{\text{CBq upon CK+CL}}{\text{CKq}} = \text{LN}$, draw it upon IL, that is, CK−CL (which is all one as if you draw CBq upon CKq−CLq by 11 *Ch.* 18.) and there ſhall ariſe $\frac{\text{CBq}\times\text{CKq}-\text{CBq}\times\text{CLq}}{\text{CKq}} = \text{LEq}$, by 13 *Lib.* 1. *Conic. Apollon.* Therefore $\sqrt{q} : \frac{\text{CBq}\times\text{CLq}}{\text{CBq}-\text{LEq}} = \text{CK}$ the *Semiaxe*; that is, $\frac{\text{CB}\times\text{CL}}{\text{OP}} =$ CK.



Which Theorem is expreſſed in Words thus, If the Square of the ſemi-diameter of the middle of the Tun be drawn into the Square of half the length, and the Product be divided by the Difference between the ſquares of the ſemi-diameters of the middle and baſe: the ſquare-root of the Quotient ſhall be the *Semiaxe* of the Spheroides.

Geometrically thus, draw EO parallel to the *Axe*; and on the ſemi-diameter CP=CB make an arch cutting EO at P; extend CP until it concur with the baſe LE produced at F. Then CF ſhall be equal to the *Semiaxe* ſought. Otherwiſe, becauſe CP=CB: and CO=LF: there will be (√u: CBq—LEq) OP. CB :: CL. CF=CK.

Conſectary. From hence it appears that the Meridians in the *Analemma* are true *Ellipſes*. For Example, ſuppoſe CIFQ a Quadrant of the *Analemma*, in which is deſcribed an *Ellipſis* IEB, I ſay the ſame is a Meridian; for ſince CQ is a Quadrant of the Æquinoctial, and FL a Quadrant of its parallel: and that 'tis the property of the Meridians to cut the Æquinoctial, and all its parallel Circles into like Segments by 10 *Lib.* 2 *Theod. de ſphæra.* If therefore it appear that CQ. CB :: LF. LE: the *Ellipſis* IEB cutting them ſhall be a Meridian. But CF=CQ and CP=CB and OC=LE. And CF. CP :: LF. OC. Therefore,

Prob. 24. BC the Hypotenuſe of a Rectangled Triangle, and CM a mean Proportional between the Baſe and the Cathetus being given, to find out the Triangle.

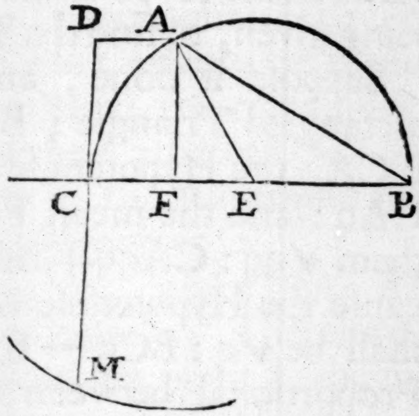
Suppoſe it done, and let BAC be the rectangled Triangle; becauſe the Baſis is BA the Cathetus ſhall be √qBCq—BAq: and the Rectangle under them √qBCq×BAq—BAqq: The Square-root whereof is √qq: BCq×BAq—BAqq: the mean Proportional between the Baſe and the Cathetus.

Again, becauſe the Cathetus is CA, the Baſe ſhall be √q: BCq—CAq: and the Rectangle under them, √q: BCq×CAq—CAqq: the Square-root whereof is √qq: BCq×CAq—CAqq the mean Proportional between the Baſe and the Cathetus. Wherefore

BCq×BAq—BAqq=CMqq, and
BCq×CAq—CAqq=CMqq.

Therefore by 9 *Chap.* 16.

$$\tfrac{1}{2}BCq \pm \surd q : \tfrac{1}{4}BCqq - CMqq : = \left\{ \begin{array}{l} BAq \\ CAq \end{array} \right.$$



In Words thus, if to half the ſquare of the Hypotenuſe be added the Square-root of the Exceſs of a quarter of the Quadrato-quadrate of the Hypotenuſe above the Quadrato-quadrate of the mean proportional between the Baſe and the Cathetus; the Aggregate ſhall be the Square of the Baſe; but if the ſame be deducted from it, the Remainder ſhall be the ſquare of the Cathetus.

Geometrically thus, on the Diameter BC and Center E, the middle point thereof, deſcribe a ſemi-circle, then make BC. CM :: CM. CD$=$AF a perpendicular within the ſemi circle. Therefore $BC \times AF = CMq$. make up the Triangle BAC. for $\tfrac{1}{4}BCq(AEq) - AFq = EFq$. Wherefore $\tfrac{1}{2}BC \pm (EF) \surd q : \tfrac{1}{4}BCq - AFq : = \begin{array}{c} BF \\ CF \end{array}$. Draw all upon BC and there will be $\tfrac{1}{2}BCq \pm \surd q : \tfrac{1}{4}BCqq - (BCq \times AFq)\ CMqq = \left\{ \begin{array}{l} BC \times BF = BAq \\ BC \times CF = CAq. \end{array} \right.$

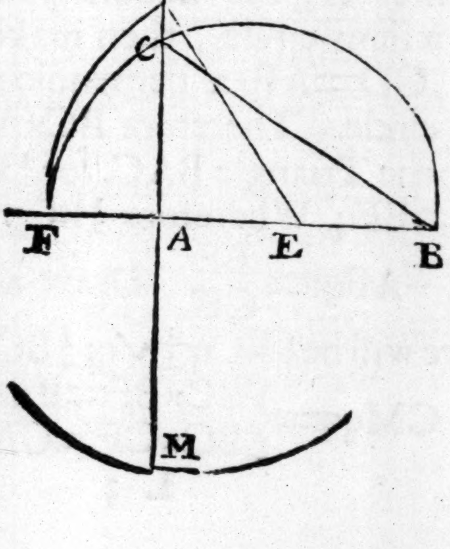
Prob. 25. BA, the Baſe of a rectangled Triangle, and AM, a mean Proportional between the Hypotenuſe and the Cathetus, being given, to find the Triangle.

Suppoſe it done, and let BAC be the rectangled Triangle; Becauſe the Cathetus is CA, the Hypotenuſe will be $\surd q : BAq + CAq :$ and the mean Proportional between them $\surd qq : CAqq + BAq \times CAq$. Alſo becauſe the Hypotenuſe is BC, the Cathetus ſhall be $\surd q : BCq - BAq$, and the mean Proportional between them $\surd q : BCqq - BAq \times BCq$. Wherefore

$$CAqq + BAq \times CAq = AMqq. \text{ and}$$
$$BCqq - BAq \times CAq = AMqq.$$

Therefore by 9 *Chap* 16.

$$\surd q : \tfrac{1}{4}BAqq + AMqq : \mp \tfrac{1}{2}BAq = \begin{cases} CAq \\ BCq \end{cases}$$

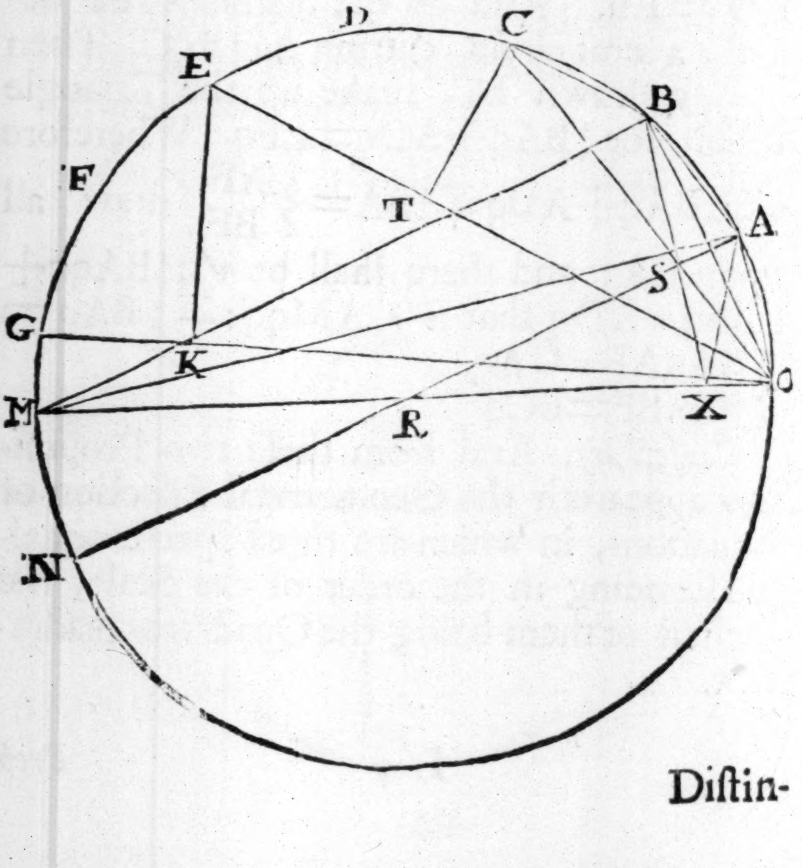


In Words thus, if from the ſquare root of the Summ of a quarter of the Quadrato-quadrate of the Baſe, and of the Quadrato-quadrate of a mean Proportional between the Hypotenuſe and the Cathetus be taken the half-ſquare of the Baſe; the remainder ſhall be the ſquare of the Cathetus; but if the ſame be added to it, the Aggregate ſhall be the ſquare of the Hypotenuſe.

Geometrically thus, make $BA.\ AM :: AM.\ AD.$ perpendicularly. Therefore $BA \times AD = MAq.$ From the point E the middle of the Baſe to the perpendicular AD, draw $ED = EF.$ And on the diameter BF deſcribe a ſemi-circle, cutting AD in C. Then having drawn BC, make up the Triangle BAC. for $\frac{1}{4}BAq + ADq = EFq.$ Wherefore $\surd q{:}\frac{1}{4}BAq + ADq{:} \mp \frac{1}{2}BA = \left\{\begin{matrix} AF \\ BF \end{matrix}\right.$ draw all upon BA: and there ſhall be $\surd q\frac{1}{4}BAqq +$ ($BAq \times ADq$ that is) $AMqq{:} \mp \frac{1}{2}BAq =$
$$\left\{\begin{matrix} BA \times AF = CAq. \\ BA \times BF = BCq. \end{matrix}\right.$$

Conſectary. And from theſe two Propoſitios appeareth the Geometrical Effection of Equations, in which are three Species equally aſcending in the order of the Scale, the higheſt of them being the Quadrato-quadratick.

Prob. 26. I ſhall alſo add a little concerning the biſection, triſection, and quinquiſection of Angles or Circumferences, to ſet forth the Excellency and admirable uſe of this Art; their geometrical Praxis is not yet found, nor the *Meſolabium*: Nevertheleſs, in *Sect.* 15. *Chap.* 18, I have touched upon ſome Cubick Equations; A diligent skilful *Algebriſt* may find innumerable others, by help whereof the *Meſolabium*, which hitherto hath lain in Darkneſs, may paradventure at length be diſcovered.



Diſtinguiſh, in a Circumference, ſeven equal parts, from O the end of the Diameter, with Letters A, B, C, D, E, F, G; draw Subtenſes alſo as in the Scheme. Take MX =MB; draw AX and XB; and a diameter NRA; and OE a perpendicular to CT, and EK a perpendicular to OG. Becauſe by 17 *Chap.* 18, *Theor.* 1. AB=AX: The Triangles BMX, ORA, OAX are like; and therefore $\frac{OAq}{Rad.} = OX$. likewiſe the Triangles OAB, ARM are like; and by 47 *è* 1 MA=√q: 4 Rad: q—OAq.

Theſe things thus premiſed, there ſhall be RA. MA, that is, Rad. √q: 4 Rad q − OAq :: OA. OB. Therefore $\frac{4Rad\mathit{q}\times OAq - OAqq}{Rad\ q}$ =OBq: which is the duplication of an angle.

And $4Radq\times OAq - OAqq = Radq\times OBq$: which is the biſection of an angle.

Then becauſe OS=OA. and SA=OX. and NS = MX = MB. There ſhall be by 17 *Chap.* 18. *Theor.* 16. $\frac{NS\times SA}{OS} = SC$: that is

$2\,Rad - \frac{OAq}{Rad}$ upon $\frac{OAq}{Rad}$ divided by OA

or $\frac{2\,Radq\times OA - OAc}{Rad\ q} = SC$. and if OA be added, then there ſhall be $\frac{3\,\text{Rad}\,q \times OA - OAc}{\text{Rad}\,q} = OC$: which is the triplication of an angle.

And $3\,\text{Rad}\,q \times OA - OAc = \text{Rad}\,q \times OC$: which is the triſection of an angle.

Alſo becauſe $2\,ET + CB = OE$. and MO. MB :: OC. OT: that is, 2 Rad. $\frac{2\,\text{Rad} - OAq}{\text{Rad}}$ :: $\frac{3\,\text{Rad}\,q \times OA - OAc}{\text{Rad}\,q}$. $\frac{6\,\text{Rad}\,qq \times OA - 5\,\text{Rad}\,q \times OAc + OAqc}{2\,\text{Rad}\,qq}$: out of the double whereof if OA be taken, there ſhall remain, $\frac{5\,\text{Rad}\,qq \times OA - 5\,\text{Rad}\,q \times OAc + OAqc}{\text{Radqq}} = OE$: which is the quintuplation of an angle.

And $OAqc - 5\,\text{Rad}\,q \times OAc + 5\,\text{Rad}\,qq \times OA = \text{Rad}\,qq \times OE$: which is the quinquiſection of an angle.

And after this manner we may proceed to find the ſeptiſection, namely $7\,Rcc \times OA - 14\,Rqq \times OAc + 7\,Rq \times OAqc + OAqqc = Rcc \times OG$. For MO. MB :: OE. OK. and $2\,OK - OC = OG$. The operation I leave to the Studious.

Now becauſe the *Radius* is put firſt, which in Multiplication and Diviſion alters nothing,

thing, therefore in all theſe Equations the *Radius* with all its powers may be omitted.

But by what artifice thoſe elaborate Equations (in which there are not only three ſpecies equally aſcending in the order of the Scale) may be reſolved, tho it belong not to my preſent deſign to teach: yet what I have written many years ago, for the uſe of Right honourable and Learned Lord *Aungier* and Baron of *Longford*, I ſhall willingly, with all poſſible brevity, communicate for the benefit of the Students in the Mathematicks.

Soli Deo Gloria.

NOTES.

Probl. 14.] *After the Solution of this Problem, he mentioneth another Caſe of it, and alſo propoſeth another Problem: For the other Caſe he gives the Theorem, but forbears the Inveſtigation of it, and omits the Solution of the other*

other Problem. Both of theſe I will here ſet down at large.

For that Caſe of this Problem, when the Exceſs is in the greater Side thus.

1. $BF.\ BK :: BD.\ \frac{BD\times BK}{BF}=BG.$

2. $\frac{BD\times BK-BFq}{BF}=FG.$

3. $\frac{BD\times BK-BFq}{2BF}=FC.$ Add BF, and there will be

4. $\frac{BD\times BK+BFq}{2BF}=BC.$ And

5. $-BD\frac{+BD\times BK+BFq}{2BF}=CL,$ *becauſe* $BC-BD=CL.$

6. $BD\times BK-2BF\times BD+BFq=2BF\times CL.$

7. $BD\times BK-2BF\times BD=2BF\times CL-BFq.$

Therefore,

8. $BK-2BF.\ 2CL-BF :: BF.\ BD :: BK.\ BG.$

For the other Problem, in which BG. BK. CL. *are given, to find, &c. Thus,*

1. $BG.\ BD :: BK.\ \frac{BD\times BK}{BG}=BF.$

2. $\frac{BGq-BD\times BK}{BG}=FG.$

3. $\frac{BGq - BD \times BK}{2BG} = FC.$

4. $\frac{BD \times BK + BGq}{2BG} = BC.$

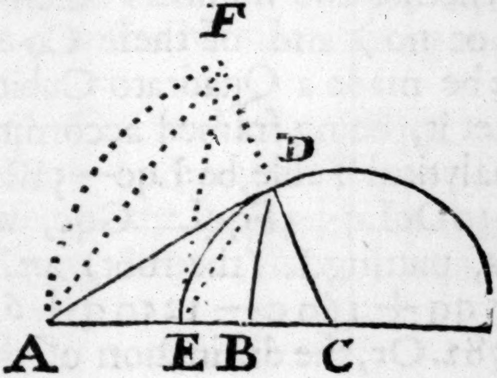
5. $\frac{2BG \times BD - BK \times BD - BGq}{2BG} = CL.$

6. $2BG \times BD - BK \times BD - BGq = 2BG \times CL.$

7. $2BG \times BD - BK \times BD = 2BG \times CL + BGq.$

8. *Therefore,* $2BG - BK . 2CL + BG : : BG . BD : : BK . BF.$

Prob. 22.] *It appears, that* ADC *and* DBC *are like Triangles, thus,* FEC *is the ſame Triangle with* ADC. *Now then, in the Triangles* AFE, EDB, *the Angle* FAE=DEB. *And about the Angle* FEA, *and* BDE *there are proportional Sides,* FE. AE : : DB. BE. (*For 'tis ſuppoſed, that* AD. (FE) DB : : AE. BE.) *And the Angles* AFE, *and* EDB *are Homogeneal. Therefore the Angle* FEA=*Angle* DBE, *and therefore* DBC, *and* FEC, *or* ADC *are like Triangles.*



OF

OF THE
RESOLUTION
OF
Affected Equations
IN
NUMBERS.

1. THE manner of conſtructing an affected Equation. Take, as you pleaſe, for B, 3: for Cq 16: for Dc, 125: for Fqq, 1296, *&c.* (it is no matter whether the numbers taken be truly figurate or no) and of theſe Co-efficients let there be made a Quadrato-Cubick Equation. Let it, being framed according to the latter analytical Table, be $Lqc - 5BLqq + 10CqLc - 10DcLq + 5FqqL = Gqc$, which in Numbers, putting L (the root) 47, ſhall be $1qc - 15qq + 160c - 1250q + 6480L = 170304782$. Or, the diſtinction of the *unciæ*

being omitted, for 15qq ſay BLqq ; for 160c ſay CqLc; for 1250q ſay DcLq ; and for 6480l ſay FqqL : For if L be 47, then is Lq=2209 : and Lc=103823 : and Lqq=4879681 : and Lqc=229345007.

The Practice of this Conſtruction.

BLqq	229345007	Lqc
15×4879681	−73195215	
CqLc	156149792	
160×103823	+16611680	
DcLq	172761472	
1250×2209	−2761250	
FqqL	170000222	
6480×47	+ 304560	
	170304782	Gqc

2. Let there be propoſed any Equation whatſoever, ſupppoſe that now found,
1qc − 15qq + 160c − 1250q + 6480l = 170304782 ; or the Numbers being changed into Symbols,
Lqc—BLqq+CqLc—DcLq+FqqL=Gqc.

And if there were more Species of Affections, conſequently they might be expreſſed by Hcc, Kqqc, Mqcc, Nccc, and ſo on further.

3. Of the Root L to be found out of theſe, there are two parts; namely, A the firſt ſide and E the ſecond, or any ſide following. Wherefore $L=A+E$. and all the powers of L are equal to the like powers of $A+E$: as $Lq=Aq+2AE+Eq$: and $Lc=Ac+3AqE+3AEq+Ec$, *&c.*

He therefore, which deſires to learn this numerous Reſolution of affected Equations, ought to be very well verſed in the *Geneſis* and *Analyſis* of pure Powers.

4. In the propoſed Equation, the *Poteſtas reſolvenda*, is *viz.* 170304782, or Gqc, is Quadrato-cubick: of which kind alſo are all the other ſeveral ſpecies of affections. For Heterogeneals can neither be added one to another, nor ſubſtracted one from another.

5. Wherefore in the ſeveral Affections two things are to be conſidered; the degree of the Affection and the Co-efficient. As in 15qq the degree of Affection is quadrato-quadratick, and the Co-efficient 15 is lateral: in 160c the degree of Affection is cubick, and the Co-efficient 160 is quadratick: in 1250q, the degree of Affection is quadratick, and the Coefficient 1250 cubick. Laſtly, in 6480l the degree of Affection is lateral, and the Coefficient 6480 quadrato-quadratick: as will evidently appear by comparing both Deſignations

the Deſignations of that Equation. And from hence ariſe two Conſectaries, for the Extraction of ſingle Roots.

6. The firſt Conſectary is, if the Root of the Co-efficient according to its own kind, drawn upon the degree of Affection, be multiplied upon the Co-efficient: the product ſhall be of the ſame kind with the poteſtas Reſolvend: as in the fore-going Equation, if the ſide 15 multiplied quadrato-quadratickly be drawn upon 15; and if √q160 cubed be drawn upon the Square 160; and if the √c 1250 ſquared be drawn upon the Cube 1250; laſtly, if √qq 6480 be drawn upon the quadrato-quadrate 6480; out of every one of theſe Multiplications ſhall ariſe a quadrato-cubick Number.

And this analytical Multiplication is the way of reducing any Coefficient to the Species of the (Power to be reſolved or) *Poteſtas Reſolvenda*, moſt uſual in the Extraction of the firſt ſingle Root.

7. Whence alſo it clearly appears, that if the Number ariſing out of the Coefficients reduced after this manner and compared, be leſs than the *Poteſtas Reſolvenda*, its ſide alſo is leſs than the ſide of the *Poteſtas Reſolvenda*, but if greater, 'tis greater, and if equal, equal. Therefore in this Equation, 1qc—15qq+160c—1250q+6480l=170304782 or 170304782+15qq—160c+1250q—

6480l=1qc, if both the lateral co-efficient 15 and alſo √q 160, and alſo √c 1250, and laſtly, √qq6480, be Quadrato-Cubed, there ſhall come forth four Homogene Species of Affections; namely 7593.., 3238.., 1450.:, 0581.., which indeed is eaſily done by Logarithims, and accurately enough for the purpoſe. The manner of the Operation is to be fetched out of the end of this Treatiſe, where ſome Rules are delivered concerning Logarithms. (See *Sect.* 27. together with the end of this Treatiſe.

	Logarithems.	*Coefficients:*
1) 2) 3) 4)	the Dimenſions in the	Coefficents
1)	5x1, 17609	1599
	5, 88045	7593..
2)	2, 20412	160c
	5x1, 10206. 12 | 65	
	5, 51030	—3238..
3)	3, 09691	1250q
	5x1, 03230. 10 | 8	
	5, 16150	—1450
4)	3, 81157	6480l
	5x0, 95289. 8 | 97	—0581..
	4, 76445	

In the Equation propoſed, the Species, according to their reſpective Signs, being gathered into one Sum, there ſhal be

$$170304700+759300-323800+145000$$
$$-058100=1qc=170827100.$$

Which may be done in like manner in other Equations.

8. The ſecond Conſectary is; If the Poteſtas Reſolvenda be divided by the Coefficient, the Quotient ſhall be referred to the ſame degree of Affection; that is, the Quotient ſhall be a ſide, if the Affection be under a ſide; or a Square, if under a Square; and ſo of the other degrees.

As in the former Eqaation. If 170304782 be divided by 15, the Quotient ſhall be Quadrato-quadratick; if by 160, the Quotient ſhall be Cubick; if by 1250 the Quotient ſhall be Quadratick; if laſt of all, by 6480, the Quotient ſhall be Lateral. Whereſore not always the Quotient, but moſt commonly its ſide, according to the degree of Affection, ſhall be the ſingle ſide to be extracted.

9. In the Inveſtigation alſo of the ſecond ſide this muſt be obſerved, that as the number of Figures in the Quotient is, ſo is its degree moſt commonly to be valued. As

if the Quotient conſiſt of one Figure only, 'tis a ſide; if of two, a Square; if three, a Cube, *&c.* And if the Quotient exceed 5, or 50, or 500, *&c.* It may perhaps be extended to the following Degree, eſpecially in greater Affections. And theſe are the Laws of Analytical Diviſion.

10. Neither will it be neceſſary in this Multiplication and Diviſion, to run through the whole Poteſtas Reſolvend with the whole Coefficient, but only as far as the next congruent Point.

11. For in the Reſolution of affected Equations, all the punctations of the Degrees muſt be made in the Poteſtas Reſolvend; as they are in the Pure, thoſe of the higheſt degrees above, and thoſe of the other degrees beneath.

The Coefficients alſo, each for their Species reſpectively are to be pointed. The Punctations of the former Example ſhall be thus,

$$1qc - 15qq + 160c - 1250q + 06410l$$
$$= 170304782$$

12. The Number of Points (eſpecially if the Coefficient be negative) ſhall regularly

in each kind be equal. Wherefore if the Poteſtas Reſolvend have more or fewer Points above it, than the Coefficient hath, let ſo many Noughts be ſet before the defective Number, that the Points in each may be equal. And in extracting the ſeveral Sides, that Point of the Coefficient which is proper to that ſide, is to be ſet under the like Point of the Poteſtas Reſolvend: Which ſhall be well done, if the Unites place in the Coefficient be orderly ſet back, unto the lower Points of the Poteſtas, anſwering to the degree.

13. If any Coefficient be a Fraction, or a ſurd Side, let it be reduced to Integers with Decimal Parts.

14. And if it be needful to continue the extraction of the Root in Decimal Parts; you ſhall ſet down after the ſeparating Line, ſo many Noughts as ſhall be thought requiſite, marking them in like manner with Points above and beneath.

15. A Table ſhewing both the Diviſors and Gnomons, for the finding out the ſingle Sides in affected Equations, collected and to be continued out of the latter Analytical Table.

And note, that all the Species of every Coefficient are affirmative, if the Coefficient itſelf be affirmative, and negative, if that be negative.

For the firſt Side	For the ſingle Sides following, to make up the *Gnomon.*					
Aq	2AE .	Eq				=Cq
BA	BE .					
AC	3AqE .	3AEq .	Ec			=Dc
BAq	B2 AE .	BEq .				
CqA	CqE .					
Aqq	4AcE .	6AqEq .	4AEc .	Eqq		=Fqq
BAc	B3AqE .	B3AEq .	BEc .			
Cq Aq	Cq2AE .	CqEq .				
DcA	DcE .					
Aqc	5AqqE .	10AcEq .	10AqEc .	5AEqq .	Eqc	=Gqc
BAqq	B4AcE .	B6AqEq .	B4AEc .	BEqq .		
CqAc	Cq3AqE.	Cq3AE .	CqEc .			
DcAq	Dc2AE .	DcEq .				
FqqA	FqqE .				&c.	

16. The Diviſors are every where taken out of thoſe Things which are given in Meaſure, being diſpoſed in their juſt Order, and added together, a regard being had to their Signs.

17. If the higheſt Power of any Equation be negative, that Equation is ambiguous.

18. The firſt ſingle Side may be extracted by theſe Rules, taken out of the two Conſectaries, *Sect.* 6. and 8.

Firſt, If the Coefficient, ſet in its right place, ſtand ſo far backward that it ſcarce reacheth to the firſt point of the Poteſtas Reſolvend; nor, being analytically reduced, make any notable mutation therein, it may in the extraction of the firſt ſingle Side be quite neglected.

Secondly, If the Coefficient ſet in its right place, ſtand forth very forward, and be affirmative, it is to be devolved from point to point, until there be place for Diviſion. By which Diviſion, the Quotient found, ſhall be referred to the degree of Affection. Which is alſo to be underſtood in the extraction of the leſſer Root of an ambiguous Equation.

Thirdly, But if the Coefficient be Negative, and conſiſt of more Points than the Poteſtas Reſolvend, let the deficient places be ſupplied with Noughts prefixed, and for the firſt ſingle Side, let the Root of the Coefficient, according to the kind thereof, be taken.

Fourthly, If on both Sides the Points be equal , and the numbers in the firſt Point, both of the Coefficient and the Poteſtas Reſolvend, do not much differ. The Coefficient being reduced (by Analytical Multiplication) unto the rank of the Poteſtas, by its Root of that kind for which it is pointed, extracted under the congruent Point; if it be negative, is to be added to the Poteſtas Reſolvend; if it be affirmative, is to be ſubducted from it. For if $Ac \pm CqA = Dc$, then $Ac = Dc \mp CqA$. But if the greater Side of an ambiguous Equation be ſought, the Poteſtas Reſolvend is to be ſubducted out of the Coefficient reduced. for if there be $CqA - Ac = Dc$

there will be $Ac = CpA - Dc$.

Then the Root of the Sum or Difference ſhall be the firſt Root to be extracted. And note, that the greater Side of an ambiguous Equation is found out ſometimes by Diviſion ; ſometimes by extraction of the Root out of the Coefficient, but moſt commonly by Reduction of the Coefficient.

19. And theſe Precepts being diligently conſidered, that at length ſhall be the firſt true ſingle Side, which firſt of all afford-

of

eth ſuch a Diagonal; which, together with the Coefficients multiplied, as the condition of the Equation requires, according to the precedent Table; and all being gathered into Sum, (diligent regard being had to the Signs and Seats, or Places,) ſhall bring forth a number not greater than the Poteſtas Reſolvend, from which it is to be ſubducted.

And it is to be noted, that a negative Number, how great ſoever, is leſs than every both affirmative, and leſſer negative. As —4 is leſs than 1, and than —1. Alſo 'tis to be noted, that Subduction changeth the Sign of the Number to be ſubducted: As out of 4 take 6, there remains 4−6; that is, —2. And out of —4 take —6, their remains —4+6, that is 2. Laſtly, out of 4 take —6, their remains 4+6, that is 10. Wherefore in the extraction of the firſt ſingle Side, ſome tryals muſt be made until you find out the true ſide; which you ſhall certainly know by the next greater.

20. In the conſtitution of the Diviſor, for finding out the ſecond Side; the ſeat of the Coefficient being drawn into any degree, muſt be ordered according to the Punctation of its proper Degree; that is, the ſeat of the Coefficient under a ſide, ſhall be diſtant one place towards the left Hand,

from the Point or Seat of the Coefficient it ſelf; the Seat of the Coefficient under a Square two places, under a Cube, three, *&c.* And to avoid confuſion, it will do well in the remainder of the Poteſtas Reſolvend, to diſtinguiſh only thoſe Punctations, which ſerve for the preſent Extraction.

21. Then the ſecond ſingle Side ſhall be extracted thus. Let the Diviſors of every kind found by the precedent Table, and diſpoſed in their juſt order, be gathered into one Sum; and let the remainder of the Poteſtas Reſolvend be divided by that total Diviſor.

For the Quotient conſidered (if need be) according to the Laws of Analytical Diviſion, ſhall give the ſecond ſingle Side to be extracted.

But in this Inveſtigation many times, eſpecially if the Aggregate of the negative dividing Quantities, be almoſt equal to the Aggregate of the affirmative, ſo that the Diviſor be much leſs than the remainder of the Poteſtas Reſolvend, there will be great lubricity; which never the leſs the ingenious Analyſt will avoid.

22. Let this therefore be a perpetual Rule: That at length is the true ſecond ſingle ſide, which firſt of all affordeth a *Gnomon*, conſiſting of Complements of every kind, and Coefficients multiplied (as the Condition of the Equation ſhall require) according to the precedent Table; and all being gathered into one Sum, diligent regard being every where had both to the Signs and Seats; which Gnomon is not greater than the *Poteſtas Reſolvend*, from whence it is to be ſubducted. Wherefore trial muſt be made, until the true Root be found; which alſo by the next greater ſhall be moſt certainly known.

23. All the other ſingle ſides after the ſecond, are moſt eaſily gotten by ordinary Diviſion.

24. If the Affections be compounded of Affirmatives and Negatives, the foregoing Precepts are to be uſed mixtly with skill and judgment: and in eſtimating the ſides a greater Affection ſhall ever weigh down a leſſer. But frequent exerciſe both in the *Geneſis* and *Analyſis* of all ſorts of Powers will render theſe matters eaſie and familiar.

25. But becauſe I have ſometimes formerly ſpoke of a neceſſity of Trials, which would be troubleſom in manifold Affections, and where the degrees are ſomewhat high, I

ſhall here, to cloſe all, ſet down two ways of eaſing that Trial. One by depreſſion out of *Chap.* 16. *Sect* 7. of my *Clavis*; another by the Canon of 1000 Logarithms. But in both theſe ways, if the Equation be ambiguous, all the Signs thereof are to be changed. Here alſo 'tis to be noted, that a negative Number, how great ſoever, is leſs than every both affirmative and leſſer negative.

26. To find the ſingle ſides by depreſſion. If the firſt ſingle ſide be ſought; in every given *Species* of the Equation, cut off with a ſeparating Line all the points after the firſt. Then divide all the *Species* by the *Latus*, that is, depreſs them one degree.

Examp. I. 1qq—72c+238600l=8725815. This by depreſſion is made 1c+238|6—7 |2q=L) 872|5. If A be 4, then 4) 872 |5 (218|1, juſt.

And +64+1238|6—115|2=187|4 leſs than juſt.

If A be 5, then 5) 872|5 (174|5 juſt.

And +125+238|6—180|0=183|6; greater than juſt.

Therefore the true ſide A=5—1. that is, 4.

Examp. **II.** Of an ambiguous Equation $1c - 3257l = -45744$. This by depreſſion is made, $1q - 32|\underline{5} = L) - 45|\underline{7}$.

If A be 4, then it ſhall be $4) - 45|\underline{7}(-11|\underline{4}$ juſt.

And $+16 - 32|\underline{5} = -16|\underline{5}$, leſs than juſt.

If A be 5, then $5) - 45|\underline{7}(-9|\underline{1}$, juſt.

And $+25 - 32|\underline{5} = -7|\underline{5}$, greater than juſt.

Therefore the true ſide $A = 5 - 1$ that is 4.

If the ſecond ſingle ſide be ſought. In every ſeveral *Species* cut off all the points after the ſecond; then divide all by the Square, that is. depreſs them two degrees. As in *Examp.* 1. $1qq - 72c + 238600l = 8725815$. This by depreſſion becometh $1q + L)238600 - 72l = Q)8725815$.

If A be 47, then 2209)8725815(3949. juſt.

And $2209 + 5077 - 3384 = 3896$. leſſer than juſt.

If A be 48, then 2304)8725815(3787. juſt.

And $2304 + 4971 - 3456 = 3819$. greater than juſt.

Therefore the true ſide is, $48 - 1$, that is, 47.

27. To find the ſecond ſingle Side by Logarithms.

The Index of every Logarithm is taken out of the Table in the beginning of the *Clavis*,

vis, according to the diſtance of its firſt Figure, before or after the Unites place, the Index whereof is 0. Therefore the ſame Figures diſpoſed in the ſame Order have the ſame Logarithm. But the Indexes may be diverſe, as of the 436, the Logarithm is 2, 6394865; but of the number 43600, the Logarithm is 4, 6394865; and of the number 4|36, the Logarithm is 0, 6394865; laſtly, of the number 0|00436, the Logarithm is $\bar{3}$,6394865.

The Sum of two Logarithms is the Logarithm of the Product of their Valors: and their difference is the Logarithm of the Quotient, as becauſe $4|36\times9=39|24$, the Logarithm of it is, $1,5937290=0,6394865+0,9542425$; and becauſe $9)39|24(4|36$: the Logarithm hereof $0,6394865=1,5937290-0,9542425$: the Logarithm of the ſide drawn upon the Index or number of dimenſions of any *Poteſtas* is the Logarithm of the ſame Poteſtas. As becauſe of the number 436, the Logarithm is 2, 6394865, it ſhall be that $2,6394865\times2=$Log: Q: 436. And $2,6394865\times3=$Log: C: 436. And $2,6394865\times4=$Log: QQ. 436, *&c.*

The Logarithm of any *Poteſtas* divided by the number of its dimenſions affordeth the Logarithm of its Root.

If, in a continued *Series* of Geometrical Proportionals, the Logarithm of the firſt Term be taken out of the Logarithm of the ſecond, the remainder ſhall be the Logarithm of the Ratio. Which, if it be drawn upon the number of all the Terms wanting one, (which is the number of Ratio's) and then be increaſed with the Logarithms of the firſt Term, ſhall be the Log. of the laſt Term.

28. And let this ſuffice concerning the knowledge of Logarithms, which being underſtood, the following Examples being diligently inſpected will make the reſt of the work eaſie. In which alſo all the punctations after the firſt are to be cut off by a ſeparating Line.

Examp. I. $1qq - \dot{7}\dot{2}c + 23\dot{8}6\dot{0}0l = 87\dot{2}581\dot{5}$. juſt. let the two firſt ſingle ſides be,

$$\begin{array}{lr|l c c}
 & & & \boxed{-72} & \boxed{+238600} \\
 & & & \overline{1,85733} & \overline{5,37767} \\
47: & 1,67209 & 8 & & \\
\text{Cu.} & 5,01629 & & \underline{5,01629} & \underline{1,67210} \\
\text{QQ.} & 6,68839 & & 6,87362 & 7,04977 \\
 & +4880 & & -7475 & +11213
\end{array}$$

And $4880 + 11213 - 7475 = +8618$ leſs than juſt.

$$\begin{array}{ll|lll}
48. & 1{,}68124 & 1 & 1{,}85733 & 5{,}37767 \\
\text{Cu.} & 5{,}04372 & & \underline{5{,}04372} & \underline{1{,}68124} \\
\text{QQ.} & 6{,}72496 & & 6{,}90105 & 7{,}05891 \\
 & +5308 & & -7963 & +11455
\end{array}$$

And $5308 + 11455 - 7963 = 48800$: greater than juſt.

Therefore the true root ſhall be $48 - 1$ that is, 47.

Examp. II. $1c - 3\dot{2}5\dot{7}\,l = 4\dot{5}74\dot{4}$. juſt. Let the two firſt ſingle ſides be,

$$\begin{array}{ll|ll}
 & & & \underline{|-3257|} \\
48. & 1{,}68124 & 1 & 3{,}51282 \\
\text{Cu.} & 5{,}04372 & & \underline{1{,}68124} \\
 & +1106 & & 5{,}19406 \\
 & & & -1563
\end{array}$$

$1106 - 1563 = -457$. leſs than juſt (at leaſt not greater.)

$$\begin{array}{ll|ll}
49. & 1{,}69019 & 6 & 3{,}51282 \\
\text{Cu.} & 5{,}07059 & & \underline{1{,}69020} \\
 & 1176 & & 5{,}20302 \\
 & & & -1596
\end{array}$$

$+1176 - 1596 = -420$. greater than juſt.

Therefore the true Root ſhall be $49 - 1$, that is, 48.

The ſecond ſide may be ſought by Logarithms, depreſſion alſo preceding, as in *Examp.* V. $1qq - \underset{\cdot}{1}2\underset{\cdot}{4}^{6}|\underline{\underset{\cdot}{0}09} = 08972625\dot{5}6.$ Which by quadratical depreſſion becomes, $1q - 124^{6} = Q)\,8972|\underline{6}.$ Suppoſe the two firſt ſingle ſides.

$$\begin{array}{ll|l|}
34. & 1,53148 & 8972|\underline{6} \\
\hline
Q. & 3,06296 & 3,95537 \\
 & +1156 & 3,06296 \\
\end{array}$$

$$0,89141:\ \text{Valor } 7|77.\ \text{juſt.}$$

$$+1156 - 124^{6} = -90.\ \text{leſs than juſt.}$$

$$\begin{array}{ll|l}
36. & 1,55630 & 3,95337 \\
 & 3,11260 & \underline{3,11260} \\
 & +1296 & 0,84077:\ \text{Valor } 6|\underline{93}.\ \text{juſt.} \\
\end{array}$$

$$+1296 - 124^{6} = +50\ \text{greater than juſt.}$$

Therefore the true Root falleth between 34 and 36.

Thus have I, in XXVIII. Sections or Precepts (which is a perfect number) diſpatched the Doctrine of the *Reſolution of Affected Equations*, by the Aſſiſtance of God the giver of all good things. *His therefore be all the Praiſe, Honour, and Glory for ever.* Amen.

Some Examples of Equations *reſolved in Numbers.*

I Shall here forbear to touch the Analyſis of Quadratick Equations, and of all other, which conſiſt of three *Species* equally aſcending in the order of the Scale: becauſe in *Chap.* 16. *Sect.* 9. *Clavis*, a more eaſie way is taught than this general Method; and proceed to Example. of Equations otherwiſe affected.

Laſtly. In the end I ſhall ſet down ſome Notes, in which the Reaſon of the Operation, in the Inveſtigation of the ſingle ſides, ſhall be opened out of the former Precepts.

I will begin with the Reſolution of that Equation, the Conſtruction whereof you have in the beginning of this Treatiſe.

$$1qc - 15qq + 160c - 1250q + 6480 = 170304782.$$

that is, $Lqc - BLqq + CqLc - DcLq + FqqL = Gqc.$

Example I.

1qc — 15qq + 160c — 1250q + 06480l = 17030304782. that is, Lqc — BLqq + CqLc DcLq + FqqL = Gqc.

1703	04782	(47
1	5	— B
1	250	— Dc
1	60	Cq
	6480	Fqq
1024		Aqc
102	40	Cq Ac
2	5920	Fqq A
+ 1128	9920	
384	0	— B Aqq
20	000	— Dc Aq
— 404	000	
724	9920	*Ablatit.*
R 978	05582	
128	0	5 Aqq
6	40	10 Ac
	160	10 Aq
	20	5 A
7	680	Cq 3 Aq
	1920	Cq 3 A
	160	Cq
	6480	Fqq

+142	50040	
38	40	—B4Ac
1	440	—B6Aq
	240	—B4A
	15	—B
1	0000	—Dc2A
	1250	—Dc
—40	87665	
+101	62375	*Diviſor*
896	0	5AqqE
313	60	10AcEq
54	880	10AqEc
4	8020	5AEqq
	16807	Eqc
53	760	Cq3AqE
9	4080	Cq3AEq
	54880	CqEc
	45360	FqqE
+1333	62047	
268	80	—B4AcE
70	560	—B6AqEq
8	2320	—B4AEc
	36015	—BEqq
7	0000	—Dc2AE
	61250	—DcEq
—355	56465	
+978	05582	*Ablatit.*

Example

Example II.

1c+420000l=247651713.

That is, Lc+CqL=Dc.

247	651	713	(417
42	000	0	Cq
64			Ac
168	000	0	CqA
232	000	0	*Ablatit.*
℞ 15	651	713	
4	8		3Aq
	12		3A
4	200	00	Cq
9	120	00	*Diviſor.*
4	8		3AqE
	12		3AEq
	1		Ec
4	200	00	CqE
9	121	00	*Ablatit.*

℞ 6	530	713	
	504	3	3Aq
	1	23	3A
	420	000	Cq
	925	530	*Diviſor.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
2	940	000	CqE
6	530	713	*Ablatit.*

4	1
16	8
	1
16	81

Example III.

1c + 1007q = 247617936,
That is, Lc + BLq = Dc.

247	617	936	(417
10	07		B
64			Ac
161	12		BAq
225	12		*Ablatit.*

℞2 2	497	936	
4	8		3Aq
	12		3A
8	056		B2A
	100	7	B

13

13	076	7	*Diviſor.*
4	8		3AqE
	12		3AEq
	1		Ec
8	056		B2AE
	100	7	BEq
13	077	7	*Ablatit.*
℞ 9	420	236	
	504	3	3Aq
	1	23	3A
	825	74	2BA
	1	007	B
1	332	277	*Diviſor.*
3	530	1	3AqE
	60	27	3AEq
		343	Ec
5	780	18	B2AE
	49	343	BEq
9	420	236	*Ablatit.*

4	1
16	
	8
	1
16	81

Example IV.

1qq − 44299005l = 22252086

Lqq − DcL = Fqq.

0	2225	2086	(345
−44	2990	05	−Dc
+81			Aqq
−132	8970	15	−DcA
−51	8970	15	*Ablatit.*
= =	= =	= =	= = =
R 52	1195	3586	
10	8		4Ac
	54		6Aq
	12		4A
+1	352		
−4	4299	005	−Dc
+6	9220	995	*Diviſor.*
54	0		4AcE
13	50		6AqEq
1	500		4AE
	625		Eqq
+69	0625		
−22	1495	025	−DcE
+46	9129	975	*Ablatit.*

R 5	2065	3836	
+1	2793	7395	*Diviſor.*
+5	2065	3836	*Ablatit.*

Example V.

1qq—124600q=089726256, Lqq—CqLq =Fqq.

0	8972	6256	(354
—12	4600		—Cq
+81			Aqq
—112	1400		—CqAq
—31	1400		*Ablatit.*
R 32	0372	6256	
10	8		4Ac
	54		6Aq
	12		4A
+11	352		
7	4760	0	—Cq2A
	1246	00	—Cq
—7	6006	00	

+3

+3	7514	00	*Diviſor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+69	0625		
37	3800	0	—Cq2AE
3	1500	00	—CqEq
—40	4950	00	
+28	5675	00	*Ablatit.*
℞ 3	4697	6256	
	8489	1800	*Diviſor.*
+3,	4687	6256	*Ablatit.*

Example VI.

1qq—340c=621066096

Lqq—BLc=Fqq.

6	2106	1096	(354
—3	40		—B
+81			Aqq
—91	80		BAc
—10	80		*Ablatit.*

R 17	0100	6096	
10	8		4Ac
	54		6Aq
	—12		4A
+11	352		
9	180		—B3Aq
	3060		—B3A
	34	0	—B
—9	4894	0	
+1	8626	0	*Divisor.*
54	0		4AcE
13	50		6AqEq
1	500		4AEc
	625		Eqq
+69	0625		
45	900		—B3AEq
7	6500		—B3AqE
	4250	0	—BEC
—53	9750	0	
+15	0875	0	*Ablatit.*
R 1	9231	6096	
+	4692	9060	*Divisor.*
+1	9231	6096	*Ablatit.*

Example VII.

1qq—77108000l=085530576, Lqq—DcL =Fqq.

0	8553	0576	(426	
—77	1080	00	—Dc	
+256			Aqq	
—308	4320	00	—DcA	
+52	4320	00	*Ablatit.*	
R 53	2873	0576		4 . 2
25	6		4Ac	16
	96		6Aq	16
	16		4A	4
+26	576			1764
—7	7108	000	—Dc	
+18	8652	000	*Diviſor.*	
51	2		4AcE	
3	84		6AqEq	4 . . 2
	128		4AEc	1
	16		Eqq	646 8
+55	1696			948
+ 5	4216	000	+DcE	74088
+39	7480	000	*Ablatit.*	

R 13	5393	0576	
+2	2030	4080	*Diviſor.*
+13	5393	0576	*Ablatit.*

Example VIII.

3200l—1c=46577 An ambiguous Equation.
CqL—Lc=Dc.
(47 Greater Root.

46	577	
32	00	Cq
—64		—Ac
+128	00	CqA
+64	00	*Ablatit.*
R—17	423	
4	8	—3Aq
	12	—3A
—4	92	
+3	200	Cq
—1	720	*Diviſor.*
33	6	—3AqE
5	88	—3AEq
	343	—Ec
—39	823	
+22	400	CqE
—17	423	*Ablatit.*
R 00	000	

Example IX.

$$3\underset{.}{2}0\underset{.}{0}l - 1c = 46\dot{5}\underset{.}{7}\underset{.}{\dot{7}}$$

[15|7 *Leſſer Root.*

$4\dot{6}$	$57\underset{.}{\dot{7}}$	
32	00	Cq
—1		Ac
+32	00	CqA
31	00	*Ablatit.*
R 15	$57\underset{.}{\dot{7}}$	
	3	2Aq
	3	3A
—	33	
+3	200	Cq
2	870	*Diviſor.*
1	5	- 3AqE
	75	- 3AEq
	125	—Ec
—2	375	
+16	000	CqE
13	625	*Ablatit.*

R 1	952	000	
	252	05	*Diviſor.*
	745	107	*Ablatit.*
R 6	206	893	000&c.

Example X.

$53q - 1c = 13254$ An Ambiguous
$BLq - Lc = Dc$ Equation.

13	254	(49	*Greater Root.*
5	3	B	
—64		—Ac	
+84	8	BAq	
+20	8	*Ablatit.*	
R—7	546		
4	8	—3Aq	
	12	—3A	
—4	92		
4	24	B2A	
	53	B	
+4	293		

—	627	*Diviſor.*
33	6	—3 AqE
5	88	—3 AEq
	343	—Ec
—39	823	
29	68	B2AE
2	595	BEq
+32	277	
—7	546	*Ablatit.*
Ro	000	

Example XI.

53q—1c=13254

13	254	(20|06 Leſſer Root.
		B
5	3	
—8		—Ac
21	2	BAq
13	2	*Ablatit.*

R	54	000	000	
	—12	0000	0	—3Aq
		—6	00	—3A
	—12	06	00	
	21	200		B2A
		5	3	B

	+21	205	3	
	9	199	30	*Diviſor.*
	+60	000	0	—3AqE
	—	150	00	—3AEq
			+125	—Ec
	+ 60	150	125	
	106	000		B2AE
		132	5	BEq
+	106	132	5	
	45	982	375	*Ablatit.*
R	9	017	625	0001&c.

Example XII.

600341—1c=1023768

CqL—Lc=Dc.

1	023	768	(236 Greater Root.
—6	003	4	Cq
—8			—Ac
+12	006	8	CqA
+4	006	8	*Ablatit.*
R--2	983	032	
1	2		—3Aq
	6		—3A

O

+1

—1	26		
+	600	34	Cq
—	659	66	*Diviſor.*
3	6		—3AqE
	54		—3AEq
	27		—Ec
—4	167		
+1	801	02	CqE
—2	365	98	*Ablatit.*
R —	617	052	
	—96	366	*Diviſor.*
	617	052	*Ablatit.*

Example XIII.

600034l—1c=1023768

17|135 Leſſer Root.

1	023	768	
	600	34	Cq
	—1		—Ac
+	600	34	CqA
+	599	34	*Ablatit.*

R	424	428	
		3	—3Aq
		3	—3A
	—	33	
	+ 60	034	Cq
	59	704	*Diviſor*
	2	1	—3AqE
	1	47	—3AEq
		343	—Ec
	—3	913	
+	420	238	CqE
+	416	325	*Ablatit.*

R + 8	102	000	
5	916	19	*Diviſor.*
6	016	189	*Ablatit.*

R 2	086	811	000	
	591	562	57	*Diviſor.*
1	775	556	903	*Ablatit.*

R	311	254	097	000	
	59	153	637	91	*Diviſor.*
	295	767	161	625	*Ablatit.*
	R 15	486	936	375	000 &c

Example XIV.

$$1qq - 72c + 238600l = 8725815\,|\,7056$$

$$Lqq - BLc + DcL = Fqq.$$

872	5815	|7056 (47|6
−72	2	−B
+238	600	Dc
256		Aqq
954	400	DcA
+1210	400	
−406	8	BAc
+749	600	*Ablatit.*
R 122	9815	7056
25	6	4Ac
	96	6Aq
	16	4A
23	8600	Dc
+50	4360	
34	56	−B3Aq
	864	−B3A
	72	−B

−35	4312	
+15	0048	*Diviſor.*
179	2	4AcE
47	04	6AqEq
5	488	4AEc
	2401	Eqq
167	0200	DcE
+398	9881	
241	92	−B3Aq
42	336	−B3AEq
2	4696	−BEc
−286	7256	
112	2625	*Ablatit.*

R 10	7190	7056	
1	7698	808	*Diviſor.*
10	7190	7056	*Ablatit.*

Example XV. *Of Triſection.*

3l−1c=1|2586407821 00 CqL−Lc=Dc.

1	258	6 40	782	1
3	3	Cq		
	−64 4	−Ac		
+1	2	CqA		
1	136	*Ablatit.*		

(0|4469

Subtenſe.

Gr.26.

R	122	640			
	4	8	—3Aq		
		12	—3A		
	—4	92			
	+3		Cq		
	25	08	*Divisor.*		
	19	2	—3 AqE		
	1	92	—3AEq		
		64	—Ec		
	—21	184			
+	12		—CqE		
	98	816	*Ablatit.*		
R	23	824	782		
	2	417	88	*Divisor.*	
	21	665	151	*Ablatit.*	
R	2	159	631	100	
		239	506	23	*Divisor.*
	2	154	585	501	*Ablatit.*
R		05	045	599	000

Example XVI. *Of Quinquiſection.*

1qc − 5c + 5l = 1 | 147152872702092

Lqc − CqLc + FqqL = Gqc.

1	14715	28727	02092	(0|2437
+	5	Fqq		*Subtenſ.*
	−5	−Cq		*Gr.* 14.
	32	Aqc		
1	0	FqqA		
+1	00032			
	−40	CqAc		
	96032	*Ablatit.*		
R	18683	28727		
	8	0	5Aqq	
		80	10Ac	
		40	10Aq	
		10	5A	
			Fqq	
	5			
	+5008	8410		
	60		−Cq3Aq	
	30		−Cq3A	
		5	−Cq	
	−630	5		

	+4378	3410	*Diviſor.*	
	32	0	5AqE	
	4	80	10AEq	
	2	560	10AqEc	
		2560	5AEqq	
		1024	Eqc	
	20		FqqE	
+	20029	62624		
	240		−Cq3AqE	
	480		−Cq3AEq	
	32	0	CqE	
	−2912	0		
	17127	62624	*Ablatit.*	
R	1555	66103	02092	
	414	9122		*Diviſor.*
	1242	65012	09443	*Ablatit.*
R	313	01090	92649	00000

Notes on the preceding Examples.

IN the Examples of *Sect.* 26 and 28, I call that a juſt Number, which ariſeth from the Application of the *Poteſtas Reſolvend*, to the degree of the ſuppoſed Side, by which the Depreſſion is made. For this is the Meaſure unto which the other *Species* being lawfully aggregated ought to be equal. As in *Exam.* 1. of *Sect.* 26, 1c+238|6−7|2q=L)872|5. if for the firſt ſide be ſuppoſed 5: ought to be C:5:+238|6−7|2 Q:5:= 872|5 divided by 5: that is, 125+238|6−(7|2×25) 180, namely, 183|6, equal to 174|5 juſt. But it is greater, and therefore the true ſide is leſſer than 5. Therefore ſuppoſe it again 4: and try whether C:4: +238|6−7|2 Q:4: be equal to 872|5 divided by 4.

But leſt that in theſe Examples, as alſo in the following, theſe Trials be undertaken meerly at adventures, I ſhall admoniſh the Reader,

Firſt, If the Homogene *Poteſtas* of the ſide extracted exceed the *Poteſtas Reſolvend*, or if the Quantities increaſing the *Poteſtas Reſolvend* exceed thoſe that diminiſh it: the true ſide A ſhall be, for the moſt part, leſs than the ſide extracted; but if otherwiſe greater, as in this Equation,

1c

$$1c+260000l=180931713.$$

Secondly, If the Diviſors under the ſame Sign with the Remainder of the *Poteſtas Reſolvend*, exceed thoſe that are under a diverſe Sign; the true Side E ſhall be, for the moſt part, leſs than the Quotient; but if otherwiſe greater, as in this Equation,

$$15681-1c=21952.$$

The ſame alſo happens in ambiguous Equations, when the Remainder of the *Poteſtas Reſolvend*, is affirmative; as in this Equation,

$$67681l-1c=214273.$$

The Solution of theſe three Equations ſhall be ſhewed in Practiſe after the Notes.

Thirdly, If, after theſe Monits, there yet remain any doubt, the Trial ſhall moſt conveniently begin at 5, and from thence the Inquiſition be continued by odd Numbers; whethether it be done by Depreſſion, or by Logarithms.

Theſe things being premoniſhed, it remains, that we diſcuſs the Examples themſelves.

For *Example* 1. √qc1703 is 4+ by *Sect.* 18. *Rule* 1. for as appears from *Sect.* 7. in the firſt point there is no great Mutation made by the Coefficients analytically reduced. For which reaſon, the true ſide A ſhall be 4.

The true Side E is leſs than the Quotient 9, becauſe the Diviſors under the Sign + (which is the Sign of R) exceed thoſe that are under the Sign —.

For *Example* 2: 42) 247 (6— by *Sect.* 18. *Rule* 2. for 42 analytically reduced, by *Sect.* 6 and 8, becomes 252: greater than 247. And the true Side A is leſs than 6, becauſe C: 6— exceedeth 247|6.

For *Example* 3. 10)247(24+Q: 5—: by *Sect.* 18. *Rule* 2.

For *Example* 4. √c44|3 is 3+, by *Sect.* 18. *Rule* 3. Wherefore the true Side A is 3.

The true Side E is leſs than the Quotient 8—, by Monit 2.

For *Example* 5. √q12|4 is 3+, by *Sect.* 18. *Rule* 3. Wherefore the true ſide A is 3.

The true ſide is leſs than the Quotient 9—, by Monit 2.

For *Example* 6. The Lateral Coefficient 3|4 multiplied Quadrato-quadratickly, and increaſed with 62, becomes 140, 𝒬𝒬3+, by *Sect.* 18. *Rule* 4. Wherefore the true ſide A is 3.

The true ſide E, is leſs than the Quotient 9—, by Monit 2.

For *Examp.* 7. √c77 is 4, by *Sect.* 18. *Rule* 3. Wherefore the true ſide A is 4.

For *Examp.* 8. √q32 is 5|65: upon 32 is 180|8 mi 46|5; remains 144, C: 5. by *Sect.* 18. *Rul.* 4. But 144 exceeds 46|5. Wherefore the true ſide A is leſs than 5, by Monit. 1.

The true Side E is leſs than the Quotient 10, by Monit 2.

For *Example* 9. 11. 13. The Solution is eaſie by Diviſion, according to *Sect.* 18. *Rule* 3.

For *Example* 10. C: 5 : is 125, mi 13. there remains 112, C : 5— : by *Sect.* 18. *Rule* 4. But 112 exceedeth 13.

Wherefore the true Side A is leſſer than 5, by Monit 1.

The true Side E is leſs than the Quotient 12, by Monit 2.

For *Example* 12. √q6 is 2+, upon 6, becomes 12 mi 1. there remains 11. C: 2$|\underline{50}$, by *Sect.* 18. *Rule* 4. but 11 exceedeth 1. Wherefore the true ſide A is a little leſs than 2+, by Monit 1.

The true Side E is leſs than the Quotient 5−, by Monit 2.

For *Example* 14. QQ: 7$|\underline{2}$: is −2687. And √c238$|\underline{6}$ is 6$|\underline{2}$, whoſe QQ is +1480. Then −2687+1480=−1207: This added to 872, giveth 2079, QQ: 6+ by *Sect.* 18. *Rule* 4. And becauſe the Adjectitious −2687 is greater than the Ablatitious +1490. Therefore the true Side A ſhall be leſs than 6, by Monit 1.

The true Side E is leſs than the Quotient 9, by Monit 2.

For *Example* 15, 16. Becauſe in both the leſſer Root of an ambiguous Equation is ſought, and the Coefficients reduced do not hinder, the Analyſis ſhail be made by Diviſion, according to *Sect.* 18. *Rule* 1.

The Practiſe of the Example in the firſt Monit.

1c+26|0000=180931713.

180	9	(4 the ſide A
26	0	Cq

√q26 is 5, upon 26, becomes 130. Let it be taken out of 180, there remains 50, C : 3+ : which is leſs than 180. Wherefore the true ſide A is greater than 3.

The Practiſe of the former Example in the ſecond Monit.

15681−1c=21952

21	952	(28 the two firſt Sides.
15	68	Cq
−8		Ac
+31	63	CqA
+23	36	*Ablat.*
℞ −1	408	
1	2	−3Aq
	6	−3A
−1	26	
+1	568	Cq
+	308	*Diviſor*

The Sign of R is —: but — 1|26 is leſs than + 1|568. Wherefore the true ſide E is greater than the Quotient 4.

The Practiſe of the latter Example in the ſecond Practiſe.

6768I—IC=214273

(47

214	273	
67	68	Cq
—64		—Ac
+270	72	CqA
+206	72	*Ablatt.*
R +7	553	
4	8	—3Aq
	12	—3A
—4	92	
+6	768	Cq
+1	848	*Diviſor*

The Sign of R is +. But the Diviſor made up of the Degrees of a Negative is leſs than the Coefficient Diviſor Affirmative, that is, —4|92 is leſs than + 6|768. Wherefore the true Side E ſhall be greater than the Quotient 4.

In *Sect.* 27. I have briefly handled the Doctrine of Logarithms, but clearly enough, eſpecially for the three former Species of Numeration, *viz. Addition*, *Subduction*, and *Multiplication*.

The Operation in adding or ſubſtracting, if the Indexes are Affirmative, doth nothing differ from the common way of Integers, and but a little if they be Negative; as appeareth from theſe Examples.

The Invention of Fractions $\left\{\begin{array}{llll} \underline{13.} & 1,11394. & \underline{15.} & 1,17609. \\ 17. & 1,23045.\ \& & 32. & 1,50515. \\ \textit{Log.} & 1,88349. & \textit{Lo.} & 1,67194. \end{array}\right.$

Addition.

$\begin{array}{l} \text{To} \\ \underline{\text{Add}} \\ \text{Sum} \end{array}\left\{\begin{array}{l} \overline{1},88349. \\ \underline{\overline{1},67194.} \\ 1,55543. \end{array}\right.$ $\begin{array}{l} \text{Out of} \\ \underline{\text{Take}} \\ \text{Rests} \end{array}\left\{\begin{array}{l} \overline{1},88349 \\ \underline{\overline{1},67194} \\ 0,21125 \end{array}\right.$

Multiplication.

Of the Side $0|\underline{0064}$. Of the Side $\underline{6|0064}$.

$3\times\overline{3},80614$. $2\times\overline{3},80614$.

Cube $\overline{7},41824$. Square $\overline{5},61228$

The Difficulty of the Diviſion of a Logarithm having a negative Index, by 2, 3, 45, *&c.* conſiſteth in the Inveſtigation of the Index of the Quotient: For which purpoſe this Table ſerveth.

Diviſors.	2)				$\overline{1}$.	$\overline{2}$	$\overline{1}$
					$\overline{3}$.	$\overline{4}$	$\overline{2}$
					$\overline{5}$.	$\overline{6}$	$\overline{3}$
					$\overline{7}$.	$\overline{8}$	$\overline{4}$
				$\overline{1}$.	$\overline{2}$.	$\overline{3}$	$\overline{1}$
&c.	(3			$\overline{4}$.	$\overline{5}$.	$\overline{6}$	$\overline{2}$
				$\overline{7}$.	$\overline{8}$.	$\overline{9}$	$\overline{3}$
	4)		$\overline{1}$.	$\overline{2}$.	$\overline{3}$.	$\overline{4}$	$\overline{1}$
			$\overline{5}$.	$\overline{6}$.	$\overline{7}$.	$\overline{8}$	$\overline{2}$
	5)	$\overline{1}$.	$\overline{2}$.	$\overline{3}$.	$\overline{4}$.	$\overline{5}$	$\overline{1}$
		$\overline{6}$.	$\overline{7}$.	$\overline{8}$.	$\overline{9}$.	$\overline{10}$	$\overline{2}$
		40.	30.	20.	10.	0	

In this Table the Diviſors are towards the left Hand within the crooked Line.

Then towards the right Hand follow the Negative Indexes of the Logarithms to be divided.

To theſe in each ranck ſtand Collateral, the Negative Indexes of the Quotients.

But the Numbers which are written beneath. 0. 10. 20. 30. 40. ſhew the Numbers to be added to the firſt Figure of the Logarithm to be divided, whoſe Negative Index is found above in the ſame Column near the Diviſor. As, if the Logarithm $\overline{7}$,41842 be to be divided by 3, ſeek $\overline{7}$ by 3) and there ſhall be given 3 Collateral, for the Index of the Quotient; and the number 20, beneath, which added to 4, the firſt Figure of of the Dividual makes it 24, in which the Diviſor is 8 times contained.

Diviſion.

3) $\overline{7}$,41842. 2) $\overline{5}$,61228.
Side $\overline{3}$,80614. Side $\overline{3}$,80614.

THE END.

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